

Consistent Resolution of Some Relativistic Quantum Paradoxes

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Abstract

A relativistic version of the (consistent or decoherent) histories approach to quantum theory is developed on the basis of earlier work by Hartle, and used to discuss relativistic forms of the paradoxes of spherical wave packet collapse, Bohm's formulation of Einstein-Podolsky-Rosen, and Hardy's paradox. It is argued that wave function collapse is not needed for introducing probabilities into relativistic quantum mechanics, and in any case should never be thought of as a physical process. Alternative approaches to stochastic time dependence can be used to construct a physical picture of the measurement process that is less misleading than collapse models. In particular, one can employ a coarse-grained but fully quantum mechanical description in which particles move along trajectories, with behavior under Lorentz transformations the same as in classical relativistic physics, and detectors are triggered by particles reaching them along such trajectories. States entangled between spacelike separate regions are also legitimate quantum descriptions, and can be consistently handled by the formalism presented here. The paradoxes in question arise because of using modes of reasoning which, while correct for classical physics, are inconsistent with the mathematical structure of quantum theory, and are resolved (or tamed) by using a proper quantum analysis. In particular, there is no need to invoke, nor any evidence for, mysterious long-range superluminal influences, and thus no incompatibility, at least from this source, between relativity theory and quantum mechanics.

I Introduction

Three quarters of a century after the establishment of its basic principles the physical interpretation of nonrelativistic quantum theory remains a controversial subject. The mathematical structure of the theory, a suitable Hilbert space together with the unitary time

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evolution produced by Schrödinger's equation, is universally accepted. The controversy has to do with the meaning to be assigned to a wave function, the role of measurements, the significance of wave function collapse, the interpretation of macroscopic quantum superpositions (Schrödinger's cat), a proper understanding of entangled states — such as in the famous Einstein-Podolsky-Rosen (EPR) paradox — and similar topics [1]. While a failure to understand these matters has not prevented the application of quantum theory to an enormous range of phenomena, it does make the subject confusing and difficult for students, and for professional physicists who want to apply quantum mechanics to a new domain, such as quantum information. A good physical theory requires both a sound mathematical framework and a consistent physical interpretation, and the latter is not entirely satisfactory in current quantum mechanics textbooks.

The situation does not improve upon going from nonrelativistic to relativistic quantum mechanics and field theory. The mathematics is more elegant and harder to follow, but the same conceptual difficulties relating the mathematics to physical reality remain; indeed, they are worse. Wave function collapse, which is something of an embarrassment for the nonrelativistic theory, gives rise to serious conceptual problems in the relativistic case, and there have been numerous discussions about this problem and how to deal with it, among them [2–12]. The original EPR argument [13] was formulated without reference to relativity theory. However, the fact that quantum theory predicts violations of Bell's inequality [14,15], together with the experimental vindication of this prediction, [16,17], is nowadays often interpreted to mean that quantum mechanics is nonlocal in the sense that certain causes can produce immediate effects a long (macroscopic) distance away [7,18]. This, of course, calls into question a basic principle of relativistic physics. In addition there are other quantum paradoxes, somewhat analogous to EPR, in which Lorentz invariance is an explicit part of the construction [6,19–21], and their existence suggests some conflict, or at least a certain tension, between quantum theory and relativity.

Most paradoxes of nonrelativistic quantum mechanics are closely linked to a single fundamental difficulty which the founding fathers did not solve: introducing probabilities into the theory in a fully consistent way. Conventional textbook quantum theory, following the lead of von Neumann [22], and London and Bauer [23], employs a deterministic unitary time development based upon Schrödinger's equation, and then assumes that a *measurement* will, for some reason, have a random outcome whose probability can be calculated, even though its existence cannot be justified, using Schrödinger's wave function. Assigning measurements this special role in a fundamental theory seems rather odd, and generations of students have been just as perplexed by it as were their teachers. To be sure, a bizarre idea that helps organize our experience should not be rejected out of hand, and the algorithm by which a wave function is used to calculate probabilities of measurement outcomes has been extremely fruitful, with numerous results in very good agreement with experiment. At the same time, the measurement approach has given rise to an enormous set of conceptual headaches. In the field of quantum foundations these are referred to collectively as the *measurement problem*, and there has been very little progress in solving them [24]. In short, while invoking measurements makes it possible to calculate probabilities which agree with experiment, in many other ways this approach to a fundamental understanding of quantum theory causes

more problems than it solves.

In the last two decades methods based upon the idea of quantum histories (consistent or decoherent histories) have been used to introduce probabilities into quantum theory in a consistent way without making any reference to measurement, by treating quantum dynamics as an inherently stochastic process [27–35]. This allows measurements to be thought of not as something special, but as particular instances of quantum processes to which quantum theory assigns probabilities using the same laws which apply to all other processes. The probabilities of measurement outcomes obtained in this way are identical with those computed using the older approach, and thus in complete agreement with experiment. But in the new approach measurements are no longer necessary for interpreting quantum theory, and as a consequence the measurement problem disappears. This does not mean that quantum mechanics reduces to classical physics. Instead, its seeming oddities, when properly understood, are seen to be the consequences of a perfectly consistent mathematical and logical structure, applicable to both microscopic and macroscopic systems, which differs in crucial respects from that of classical physics. In brief, quantum reality is different from classical reality, just as relativistic reality differs from (pre-relativistic) classical reality.

Introducing probabilities in a consistent way without appealing to measurements makes it possible to resolve or at least *tame* the paradoxes of nonrelativistic quantum theory, as shown in detail in Chs. 20 through 25 of [35]. The notion of taming a paradox can be illustrated by reference to the well-known twin paradox of relativity theory. Intuitively it seems surprising that the astronaut who has been traveling for many years at high speed returns to earth biologically much younger than his stay-at-home twin brother. But (special) relativity provides a consistent framework which allows us to understand, in both mathematical and physical terms, why this can be so. This explanation does not, and should not, remove our surprise when we first encounter the difference between the relativistic idea of time and the notion of absolute time that seems much closer to our everyday experience. However, once we understand relativistic principles the twin paradox is no longer a conceptual headache, an unsolved mystery that calls into question our understanding of physical reality. Instead, it is a striking illustration of how that reality differs from what we naively expected before studying it more closely.

The goal of the present paper is to apply the same approach, probabilities not based on measurement, to relativistic versions of the nonrelativistic paradoxes which have been successfully tamed by this method, in particular, to relativistic versions of wave function collapse, EPR, and Hardy’s paradox. Before presenting a brief outline of the rest of the paper, it is worth noting that there are numerous conceptual difficulties and paradoxes of relativistic quantum mechanics and quantum field theory which are not addressed in the present paper. While there is no need to list all of them, one in particular is worth mentioning: the problem of *microlocality*, to be distinguished (or so we believe) from that of the *macrolocality* needed for discussing the paradoxes just mentioned. Microlocality is associated, at least intuitively, with the idea that relativistic quantum particles cannot be well localized in regions with linear dimensions which are too small, nor precisely localized, in a sense which would please a mathematician, in any finite region. For example, to take the physicist’s point of view, it does not make sense to think of an electron localized in a region smaller than its Compton

wavelength. Microlocality comes up in Newton-Wigner states, and in Hegerfeldt's results on nonlocalization; see [36–38] for some representative literature. The present paper contains no attempt to resolve the mysteries of microlocalization; instead the strategy, as in [3, 4], is to avoid them, by setting up relativistic quantum histories using a coarse-grained length scale: distances which, though not necessarily macroscopic, are always significantly larger than the relevant Compton or other length scale which might limit the notion of locality employed in Sec. III B. The resulting formulation can at best be a good approximation, but we believe it is still sufficient for taming those paradoxes with which we are concerned, for they involve quantum correlations which can exist over length scales of centimeters or even, in the case of light, meters or kilometers. Thus we take the attitude that the problems and paradoxes of macrolocality can be separated from issues of relativistic microlocality. Should this be false it would, needless to say, call into question the main results of this paper. (Note that the histories approach has been applied to some microlocal problems by Omnès [39].)

In order to make the present work self-contained, Sec. II contains a summary of the essential ideas of the nonrelativistic histories approach as formulated in [35], and a specific example is considered in Sec. II D, to make the presentation a bit less abstract. (Here, and later, we omit the arguments needed to show that various families of histories are consistent or inconsistent, as they are not needed in order to follow the presentation. A detailed discussion of consistency conditions and methods for checking them will be found in Chs. 10 and 11 of [35]; a more compact presentation is in [32].) The formulation of relativistic histories presented in Sec. III follows in the footsteps of earlier work by Hartle [40]. Most of the ideas are not new, but the way in which they are presented owes something to developments in the nonrelativistic theory during the last decade. There is one important difference between our approach and Hartle's. He employed regions with a finite extent in the time direction, whereas we use spacelike hypersurfaces which at each point in space are instantaneous in time. Given that the present formulation is, as explained in the previous paragraph, coarse grained in space, there is no reason not to think of it as (in some sense) coarse grained in time, so the difference with Hartle's formulation may not be all that significant. There is other work [39, 41] which has made use of relativistic histories and it is, we believe, consistent with the present formulation in so far as they overlap.

The discussion of relativistic paradoxes begins in Sec. IV with the collapse of the wave function of a single particle emitted in a spherical wave as the result of some nuclear decay. This, or rather a one-dimensional analog which serves to illustrate the main points, is treated in some detail, for in resolving (or taming) this paradox one employs most of the ideas needed to handle relativistic versions of the Einstein-Podolsky-Rosen (EPR) paradox as formulated by Bohm, taken up in Sec. V, and a paradox due to Hardy, considered in Sec. VI. Studying these three paradoxes suffices, we believe, to expose the basic principles needed to tame other paradoxes of the same general sort, the kind which tempt one to think that the quantum world is inhabited by mysterious influences which can propagate at superluminal speeds. One of our main conclusions is that there are no such influences; belief in them seems to have arisen through confusion over the proper rules for reasoning about the physical properties of quantum systems, that is, logical difficulties which are essentially the same in both the nonrelativistic and the relativistic theory, although relativity adds a few interesting twists.

Counterfactual forms of relativistic paradoxes are, strictly speaking, outside the scope of the present paper, because analyzing them requires a relativistic generalization of the formulation of counterfactual reasoning in [42] and Ch. 19 of [35], and this is not yet available.

A concluding Sec. VII provides a summary both of the principles of relativistic histories in Sec. III and of the lessons learned through exploring and taming the paradoxes in Secs. IV to VI.

II Nonrelativistic Quantum Histories

II A Kinematics

There are by now a number of treatments of the basic principles of nonrelativistic quantum theory from a histories perspective [29,31–35]. While these differ in some details, the basic strategy is the same; in what follows we use the notation in [35], where the reader will find a detailed discussion of various points which, of necessity, are treated in a summary fashion in the present discussion.

The histories approach starts with the idea, which goes back to von Neumann, Sec. III.5 of [22], that any property of a quantum system at a given instant of time corresponds to a *subspace* of the quantum Hilbert space, and the negation of this property to the orthogonal complement of this subspace [43]. Equivalently, a property is represented by a projector P (orthogonal projection operator) onto the subspace in question, and its negation by the projector $I - P$, where I is the identity operator. Such properties cannot, in general, be combined with one another in the manner which is possible in classical physics. For example, for a spin-half particle the property that the z component of angular momentum be positive, $S_z = +1/2$ in units of \hbar , corresponds to a one-dimensional subspace in the Hilbert space, as does its counterpart $S_x = +1/2$ for the x component of angular momentum. In classical physics one would then be able to make sense of the conjunction of these two properties: $S_z = +1/2$ AND $S_x = +1/2$. But in quantum theory this is not possible, at least without altering the rules of logic as suggested by Birkhoff and von Neumann [44]: $S_z = +1/2$ AND $S_x = +1/2$ is not a meaningful proposition, as it corresponds to no subspace in the Hilbert space, nor is its negation $S_z = -1/2$ OR $S_x = -1/2$ a meaningful proposition. (For more details, see [33,35].) In the histories approach two propositions which stand in such a relationship are called *incompatible*, and the basic strategy for avoiding the contradictions associated with nonrelativistic quantum paradoxes is to insist that all valid quantum descriptions consist of compatible entities: properties, histories, etc. In particular, properties corresponding to subspaces whose projectors do not commute with each are always incompatible.

A quantum *history* is a sequence of quantum properties at a succession of times, say

$$t_0 < t_1 < t_2 < \cdots < t_f, \quad (1)$$

and has the form

$$Y^\alpha = P_0^{\alpha_0} \odot P_1^{\alpha_1} \odot P_2^{\alpha_2} \odot \cdots \odot P_f^{\alpha_f}, \quad (2)$$

where $P_j^{\alpha_j}$ is some projector representing a property of the system at the time t_j . The α_j is a label which differentiates this projector from other projectors representing alternative properties which the system might possess at this time. The collection of such projectors at time t_j form a *decomposition of the identity* $\{P_j^{\alpha_j}\}$, or

$$I_j = \sum_{\alpha_j} P_j^{\alpha_j}. \quad (3)$$

(The subscript on the identity operator I can be ignored in the nonrelativistic case, but is needed for the relativistic generalization.) The composite label $\alpha = (\alpha_0, \alpha_1, \alpha_2 \dots)$ on Y in (2) identifies the history as a whole, and the collection of all histories of this sort (for a fixed decomposition of the identity at each time) form a *sample space* of histories. Note that the superscripts in (2) and (3) are labels, not powers. This usage need not cause any confusion, since the square of a projector is the projector itself, and thus there is never any need to raise it to some power. One often considers histories with a *fixed initial state* of the form

$$Y^\alpha = \Psi_0 \odot P_1^{\alpha_1} \odot P_2^{\alpha_2} \odot \dots \odot P_f^{\alpha_f}, \quad (4)$$

with Ψ_0 a single projector (possibly onto a pure state) independent of α .

While the symbols \odot in (2) and (4) can be regarded simply as spacers, equivalent to commas, it is actually convenient to think of them as a variant of \otimes , the operator for a tensor product, so that Y^α is a projector on the Hilbert space

$$\check{\mathcal{H}} = \mathcal{H}_0 \odot \mathcal{H}_1 \odot \mathcal{H}_2 \cdots \odot \mathcal{H}_f \quad (5)$$

of histories, the tensor product of $f + 1$ copies of the Hilbert space \mathcal{H} of the system at a single time [45]. The product $Y^\alpha Y^{\bar{\alpha}}$ of two projectors of the form (2) is zero if $\alpha \neq \bar{\alpha}$, that is, if α_j is not equal to $\bar{\alpha}_j$ for some j . Projectors on $\check{\mathcal{H}}$ of the form

$$Y = \sum_{\alpha} \pi_{\alpha} Y^{\alpha}, \quad (6)$$

where each π_{α} is either 0 or 1, form a *Boolean algebra* of history projectors, all of which commute with one another. This Boolean algebra, or the sample space which generates it, is called a *family* of histories, and in the histories approach represents the *event algebra* for a probability theory.

Whereas families of histories of the form (2) or (4), with the projectors at any given time coming from a single decomposition (3) of the identity, are the simplest kind to think about, the histories formalism actually allows for much more general possibilities, see Ch. 14 of [35], which are sometimes useful. Since including these more general families in a relativistic theory gives rise to no new problems or issues, the exposition below and in Secs. III and IV is restricted to the simpler type of family based on (3). (A further generalization allowed by Isham's formalism, projectors on the history space (5) which cannot be written as a tensor product, as in (3), or as a sum of projectors which are themselves tensor products, are excluded from the present discussion, and from the relativistic generalization given below. Such histories have yet to be given any physical interpretation.)

In ordinary probability theory one assumes that one and only one of the mutually exclusive possibilities which make up a sample space (e.g., heads and tails for a tossed coin) actually occurs. Similarly, in the histories approach to quantum theory, one supposes that one and only one of the histories which make up the sample space actually takes place in a given “experimental run”. In addition, if the history Y^α for a given α is the one which actually occurs, the successive projectors in (2) are thought of as representing actual states of affairs at the times in question. Thus the histories approach, unlike textbook quantum theory, does not confine its physical interpretation to measurements or the results of measurements. Instead, measurements are physical processes to be analyzed in the same way as all other physical processes, by constructing appropriate histories of the total quantum system including the measuring apparatus. This apparatus must be treated as a quantum mechanical system, since the histories interpretation insists that everything be discussed in quantum terms without introducing classical elements (except as approximations to quantum theory). In this way the histories approach eliminates paradoxical elements of nonrelativistic quantum theory which arise out of treating measurement as a fundamental concept.

II B Dynamics

The time development of a quantum system, in the histories perspective, is fundamentally a random or stochastic process, and the deterministic, time-dependent Schrödinger equation is used as a tool to calculate the probabilities of different histories. (To be sure, the theory allows for deterministic histories in which later events follow with probability one from some initial condition. But such “unitary” histories are exceptional cases; most histories which are of interest in connection with actual laboratory experiments are not of this form.) As is well known, by integrating Schrödinger’s equation for a closed quantum system one can obtain a collection of unitary *time development operators*, denoted here by $T(t', t)$, where the times t' and t serve as labels. For a time-independent Hamiltonian H these operators can be written as

$$T(t', t) = \exp[-i(t' - t)H/\hbar]. \quad (7)$$

Whether or not H depends on the time, as long as it is Hermitian the time development operators satisfy the following conditions,

$$T(t, t) = I, \quad T(t'', t')T(t', t) = T(t'', t), \quad T(t', t) = T^\dagger(t, t') = T^{-1}(t, t'). \quad (8)$$

for all t , t' , and t'' .

Given the time development operators, a *chain* operator for the history Y^α in (2) can be defined by writing its adjoint in the form

$$K^\dagger(Y^\alpha) = P_0^{\alpha_0}T(t_0, t_1)P_1^{\alpha_1}T(t_1, t_2)P_2^{\alpha_2}T(t_2, t_3)\cdots T(t_{f-1}, t_f)P_f^{\alpha_f}, \quad (9)$$

where the projectors appear in the same order as in (9); the operator $K(Y^\alpha)$ is then a similar product with the operators in the reverse order, and the arguments of each $T(t', t)$

interchanged. The *weight* of a history is given by

$$W(Y^\alpha) = \langle K(Y^\alpha), K(Y^\alpha) \rangle, \quad (10)$$

where the operator inner product \langle, \rangle is defined by

$$\langle A, B \rangle = \text{Tr}(A^\dagger B), \quad (11)$$

assuming the trace exists. In the case of a family of histories involving just two times, t_0 and t_1 , with an initial state $|\psi\rangle$ at t_0 and a decomposition of the identity corresponding to an orthonormal basis $|\phi^\alpha\rangle$ at t_1 , the weights are given by

$$W(\Psi_0 \odot P_1^\alpha) = |\langle \phi^\alpha | T(t_1, t_0) | \psi \rangle|^2, \quad (12)$$

where $\Psi_0 = |\psi\rangle\langle\psi|$ and $P^\alpha = |\phi^\alpha\rangle\langle\phi^\alpha|$. In this case the weights correspond to the usual Born transition probabilities, and thus (10) can be thought of as a generalization of the Born rule to the case of histories involving an arbitrary number of times.

The weights defined in (10) can be combined with whatever initial information one has about the quantum system in order to assign probabilities to the various histories, in the same manner as for a classical stochastic process; see Ch. 9 in [35]. Thus, in particular, if the system is known to have been in the initial state $|\psi\rangle$, the weight in (12) give the probabilities for the history $\Psi_0 \odot P_1^\alpha$ or, equivalently, the probability that the quantum system will be in the state $|\phi^\alpha\rangle$ at t_1 , given that it was in the state $|\psi\rangle$ at t_0 .

When three or more times are involved, the histories approach imposes additional conditions. In order that a family of histories be acceptable as a possible stochastic description of a closed quantum system, so that one can assign probabilities to the different histories, the chain operators of the form (2) must be *mutually orthogonal*,

$$\langle K(Y^\alpha), K(Y^{\bar{\alpha}}) \rangle = 0 \text{ for } \alpha \neq \bar{\alpha}, \quad (13)$$

where \langle, \rangle is the operator inner product (11). These are the *consistency conditions* or *decoherence conditions*, and the left side of (13) is often referred to as a *decoherence functional*. (Various alternative consistency conditions have been proposed from time to time; the one encountered most often is that in which the real part of the operator inner product in (17) rather than the product itself is set equal to 0.) A family of histories for which (13) is satisfied is called a *consistent family* or *framework*. A meaningful description of a quantum system in physical terms is always based upon some framework.

The weights W and the consistency conditions can also be expressed in terms of Heisenberg projectors and chain operators, defined in the following way. Let t_r be some *reference time*; its value is unimportant as long as it is held fixed. Then for each projector entering a history of the form (4), let the corresponding Heisenberg projector be defined by

$$\hat{P}_j^{\alpha_j} = T(t_r, t_j) P_j^{\alpha_j} T(t_j, t_r). \quad (14)$$

The corresponding Heisenberg chain operator is

$$\hat{K}^\dagger(Y^\alpha) = \hat{P}_0^{\alpha_0} \hat{P}_1^{\alpha_1} \hat{P}_2^{\alpha_2} \cdots \hat{P}_f^{\alpha_f}, \quad (15)$$

which is (formally) simpler than (9) in that time development operators do not appear on the right side. It is then easy to check that (10) and (13) are equivalent to

$$W(Y^\alpha) = \langle \hat{K}(Y^\alpha), \hat{K}(Y^\alpha) \rangle, \quad (16)$$

$$\langle \hat{K}(Y^\alpha), \hat{K}(Y^{\bar{\alpha}}) \rangle = 0 \text{ for } \alpha \neq \bar{\alpha}. \quad (17)$$

Note that the operators on the right side on (15) do not (in general) commute with each other, and hence the order is important. Interchanging this order by using, for example, $\hat{P}_0^{\alpha_0} \hat{P}_2^{\alpha_2} \hat{P}_1^{\alpha_1}$ in place of $\hat{P}_0^{\alpha_0} \hat{P}_1^{\alpha_1} \hat{P}_2^{\alpha_2}$ for a history based on the three times $t_0 < t_1 < t_2$ will (in general) change the value of the weight in (16). Thus keeping track of the temporal order of events is important if one wants to have physically meaningful results. On the other hand, writing the projectors on the right side of (15) in reverse order, with $\hat{P}_f^{\alpha_f}$ at the left and $\hat{P}_0^{\alpha_0}$ at the right, merely replaces $\hat{K}^\dagger(Y^\alpha)$ with its adjoint $\hat{K}(Y^\alpha)$, and this does not alter $W(Y^\alpha)$ nor, if the change is made for *all* the histories in a family, does it alter the consistency conditions (17). Consequently, the histories interpretation is invariant under a reversal of the direction of time. (Note that this is quite a different issue from time-reversal invariance of the Hamiltonian, which manifests itself in properties of the unitary operators $T(t', t)$.)

II C Refinement and compatibility

Let \mathcal{F} be a family of histories based upon decompositions of the identity of the form (3) at a set of times given by (1). We shall say that a second family \mathcal{G} is a *refinement* of \mathcal{F} (or \mathcal{F} is a *coarsening* of \mathcal{G}) provided two conditions are satisfied.

1. The collection of times at which \mathcal{G} is defined includes all those at which \mathcal{F} is defined, and perhaps some additional times.

2. At each of the times for which \mathcal{F} is defined, \mathcal{G} is based upon the same decomposition of the identity (3) as \mathcal{F} , or else upon a *finer* decomposition of the identity, one in which at least one, and possibly more, of the projectors in the original decomposition has been replaced by two or more projectors which sum up to the projector which has been replaced.

These two conditions can be collapsed into a single condition if one uses the following idea. A history of the form (2) specifies certain properties at the times given in (1), and says nothing about what is happening at any other time. Now one can “extend” the history (2) to additional times without changing its physical meaning if the identity I is used as a projector for each added time, because I represents the property which is always true, and therefore its occurrence tells us nothing we did not already know. Given two families of histories which are not initially defined at the same set of times, we can always extend the histories in the manner just indicated so that we have equivalent families defined at a larger set of times, which are now the same for both families. If we allow for such an “automatic extension”, then \mathcal{G} is a refinement of \mathcal{F} if and only if at each time where the histories in both families (of extended histories) are defined, the decomposition of the identity for \mathcal{G} is the same or finer than that for \mathcal{F} . Note that according to this definition, a family \mathcal{F} is always a

refinement of itself. Also note that a refinement \mathcal{G} of a consistent family \mathcal{F} may or may not be consistent.

Two frameworks \mathcal{F} and \mathcal{F}' are said to be *compatible* provided they possess a *common refinement* which is itself a consistent family or framework. That is, there must be some family \mathcal{G} which is both a refinement of \mathcal{F} and a refinement of \mathcal{F}' , and which satisfies the consistency conditions. Since according to the definition given above, a family is always (formally) a refinement of itself, \mathcal{G} could be \mathcal{F} or \mathcal{F}' . Indeed, if one framework is a refinement of another, the two are compatible. Frameworks which are not compatible are called *incompatible*. There are two slightly different ways in which two frameworks can be incompatible. The first is that, as families, they have no common refinement: this means that at least one of the times of interest the two decompositions of the identity contain projectors which do not commute with each other. One might call this “kinematical incompatibility”. But even if a common refinement exists, it need not satisfy the consistency conditions, leading to “dynamical incompatibility.”

A central principle of histories quantum theory is the *single framework rule* (or single family, or single set rule): a quantum description must be constructed using a single consistent family, and results from two or more incompatible frameworks cannot be combined. This is an extension to histories of the principle illustrated at the beginning of Sec. II A using the x and z components of angular momentum of a spin-half particle, and in the case of kinematic incompatibility can be justified on precisely the same basis: the mathematics of the Hilbert space structure of quantum theory as interpreted by von Neumann requires, if one takes it seriously as representing physical reality, some changes in the way one thinks about that reality. Incompatibility in this sense is a quantum concept that does not arise in classical physics, and thus there is no good classical analogy for the single framework rule. Many paradoxes of nonrelativistic quantum theory involve some violation of the single framework rule (see the discussion in Chs. 20 to 25 of [35]), and the histories approach avoids these paradoxes by strictly enforcing this rule, which plays an equally important role in relativistic quantum theory.

To complete this discussion, we note that when one uses the histories approach, *wave function collapse* is completely absent from the fundamental principles of quantum theory. If one treats quantum mechanics as a stochastic theory, then various physical consequences can be worked out by using the standard tools of probability theory, in particular, by computing appropriate conditional probabilities. For example, suppose that a measurement has a probability $1/3$ to turn out one way, apparatus pointer directed to the left, and $2/3$ to turn out a different way, pointer directed to the right. If the experiment is carried out and at the end the pointer points to the left, then probability theory allows one to calculate various probabilities using “pointer points to the left” as a *condition*. Wave function collapse as seen from a histories perspective provides a way (sometimes, but not always, a useful way) to calculate certain conditional probabilities which can also be computed by alternative methods. In particular, wave function collapse is not a mysterious physical phenomenon produced by an equally mysterious measurement process. One should think of it as something which occurs in the theoretical physicist’s notebook, not in the experimental physicist’s laboratory! (In addition, we shall sometimes use the term “collapse” in a metaphorical sense to indicate

the point at which a family of histories branches, as in (43).)

II D Example using spin half

It is helpful to see how the formalism described above applies to a particular simple example, that of the spin degree of freedom of a spin-half particle in zero magnetic field, so $T(t', t) = I$, the identity operator. Suppose that the initial state at t_0 is $|z^+\rangle$ corresponding to $S_z = +1/2$ in units of \hbar , and that at later times we use a decomposition of the identity

$$I = z^+ + z^-, \quad (18)$$

where z^+ is the projector $|z^+\rangle\langle z^+|$, and z^- the projector for $S_z = -1/2$. Histories of the form (4) based on the initial state z^+ then form a family

$$\mathcal{F}_0: z^+ \odot \{z^+, z^-\} \odot \{z^+, z^-\} \odot \cdots, \quad (19)$$

in which each history begins with z^+ , followed at later times by one of the possibilities z^+ or z^- . Because $T = I$, every history has zero weight or zero probability, apart from $z^+ \odot z^+ \odot z^+ \odot \cdots$, which has probability one. In this example, and in many of those we will consider later, most histories have zero weight and only a few occur with probability greater than zero. In such cases it is convenient to employ a shorthand in which rather than listing all possible histories, as in (19), one shows only those that have positive probability, the *support* of the consistent family. In this shorthand (19) is replaced with

$$\mathcal{F}_0: z^+ \odot z^+ \odot z^+ \odot \cdots, \quad (20)$$

and there is no harm in referring to it as the “framework \mathcal{F}_0 ” in place of the more precise “support of \mathcal{F}_0 ”. (While displaying the support is usually adequate for indicating the family one has in mind, it does not always determine unambiguously the decompositions of the identity, and sometimes one has to be more specific about which histories of zero weight are to included in the family.) In the remainder of this paper we will use this shorthand without further comment.

The *unitary* family (20) in which each of the projectors (in the support) is equal to its predecessor under the unitary map produced by the time development operator is a rather special sort of quantum description. In practice one usually deals with *stochastic* frameworks, such as

$$\mathcal{F}_1: z^+ \odot \begin{cases} x^+ \odot x^+ \odot x^+ \odot \cdots, \\ x^- \odot x^- \odot x^- \odot \cdots, \end{cases} \quad (21)$$

where x^+ and x^- are projectors on the states $S_x = \pm 1/2$. The support of this consistent family consists of two histories, both having the same initial state, and each occurring with probability of 1/2. In the first history $S_z = +1/2$ at t_0 and $S_x = +1/2$ at t_1 and all later times. It is somewhat misleading to think of this history as one in which “the spin is pointing in the z direction” at t_0 and “the spin is pointing in the x direction” at t_1 and later

times, for this suggests that there is some torque acting between t_0 and t_1 to make the spin precess, whereas we are assuming there is no magnetic field present, and therefore no torque. Instead, the difference between \mathcal{F}_0 and \mathcal{F}_1 is that in the former one has chosen to describe the z component, and in the latter the x component, of spin angular momentum at times later than t_0 . A description of a classical spinning object which specifies one component, say L_z of its angular momentum at an earlier time and a different component, say L_x , at a later time tells one nothing about the direction of the total angular momentum at either time, and this is a helpful analogy in thinking about the quantum case, where $S_z = +1/2$ does *not* imply that S_x or S_y is zero.

In \mathcal{F}_1 the two histories “split”, or diverge from each other, at t_1 , but there are other frameworks in which this split occurs later, such as

$$\mathcal{F}_2: \quad z^+ \odot z^+ \odot \begin{cases} x^+ \odot x^+ \odot \cdots, \\ x^- \odot x^- \odot \cdots, \end{cases} \quad (22)$$

where it occurs at t_2 . In view of the remarks in the previous paragraph, it is evident that the presence as well as the timing of such a split — one could also call it a “collapse” — is not some sort of physical effect. Instead, it arises from the possibility of constructing various different, incompatible (in the quantum sense) stochastic descriptions of the same quantum system starting in the same initial state. This does *not* mean that one of these descriptions is correct and the others false, but rather that there is no way of combining them into a single description. This is obvious in the case of \mathcal{F}_1 and \mathcal{F}_2 because at t_1 the former assigns a value to S_x and the latter a value to S_z , whereas the Hilbert space does not allow simultaneous values of two different components of spin angular momentum. In the same way, both \mathcal{F}_1 and \mathcal{F}_2 are incompatible with \mathcal{F}_0 . The choice of which family to use in a particular circumstance is made by the physicist on the basis of what aspects of the time development he wants to discuss. If it is S_x at t_2 , then either \mathcal{F}_2 or \mathcal{F}_1 can be used, but not \mathcal{F}_0 , whereas neither \mathcal{F}_2 nor \mathcal{F}_1 can be used to describe S_z at t_2 . Also note that once a split or collapse of the kind one finds in \mathcal{F}_1 or \mathcal{F}_2 has occurred, it cannot be undone by, for example, replacing x^+ with z^+ at t_3 in both histories in (22) (or in (21)). Such a family would violate the consistency conditions, and hence not be a meaningful stochastic description of the time development of this quantum system.

One can extend this example to include measurements. Let $|X\rangle$ represent the initial state of an apparatus designed to measure S_x , and suppose that during the time interval the total system of particle plus apparatus undergoes a unitary time evolution given by:

$$|x^+\rangle \otimes |X\rangle \rightarrow |x^+\rangle \otimes |X^+\rangle, \quad |x^-\rangle \otimes |X\rangle \rightarrow |x^-\rangle \otimes |X^-\rangle. \quad (23)$$

Before and after this time both particle and apparatus remain unchanged. (One can imagine that the particle passes through the apparatus between t_1 and t_2 , but that for simplicity we have omitted the center of mass motion of the particle from our description.) It is helpful to think of $|X^+\rangle$ and $|X^-\rangle$ as macroscopically distinct apparatus states, e.g., corresponding to two positions of a visible pointer. This is an oversimplified but not misleading description of a quantum measurement; see Sec. 17.4 of [35] for a more realistic approach. (Typical laboratory

measurements or quantum systems are destructive in the sense that the measured property is significantly altered in the measurement process. The histories approach handles these without difficulty, see Ch. 17 of [35], but (23) is a nondestructive model of measurement, which makes it easier to compare with usual textbook approach.)

Let us suppose that at t_0 the combined system is in a state $z^+ \otimes X$, i.e., $S_z = +1/2$ for the particle, and the apparatus in its “ready” state. One possible framework is that of unitary time evolution of the total system:

$$\mathcal{G}_0: \Psi_0 \odot z^+ X \odot S \odot S \odot \cdots, \quad (24)$$

where the initial state is

$$\Psi_0 = z^+ X; \quad (25)$$

omitting the \otimes between the projectors z^+ and X onto the states $|z^+\rangle$ and $|X\rangle$ does not lead to any ambiguity. The projector S projects onto the state

$$|S\rangle = (|x^+\rangle|X^+\rangle + |x^-\rangle|X^-\rangle)/\sqrt{2}, \quad (26)$$

which, since the apparatus is of macroscopic size, is a *macroscopic quantum superposition* (MQS) or Schrödinger cat state. As a consequence, \mathcal{G}_0 , even though a perfectly correct quantum description of the time development, is not of much use for discussing the measurement process in physical terms. The reason is that S does not commute with either of the projectors X^+ or X^- describing the possible measurement outcomes, so if one uses the description provided by \mathcal{G}_0 it is meaningless to ascribe a position to the pointer after the measurement has taken place.

Of greater utility is the framework

$$\mathcal{G}_1: \Psi_0 \odot \begin{cases} x^+ X \odot x^+ X^+ \odot x^+ X^+ \odot \cdots, \\ x^- X \odot x^- X^- \odot x^- X^- \odot \cdots, \end{cases} \quad (27)$$

which is the measurement counterpart of \mathcal{F}_1 in (21). In this family the apparatus is in its ready state X and the particle is in one of the two states $S_x = \pm 1/2$ at t_1 . At t_2 and later times the state X^\pm of the apparatus reflects the earlier state of the particle, as one would expect given (23). From the measurement outcome X^+ at any time after t_2 , one can infer (conditional probability equal to 1) that $S_x = +1/2$ both before and after the measurement; similarly X^- implies $S_x = -1/2$ at earlier as well as later times.

The measurement counterpart of \mathcal{F}_2 is the framework

$$\mathcal{G}_2: \Psi_0 \odot z^+ X \odot \begin{cases} x^+ X \odot x^+ X^+ \odot \cdots, \\ x^- X \odot x^- X^- \odot \cdots. \end{cases} \quad (28)$$

It corresponds fairly closely to the traditional “collapse” picture of the measurement process found in textbooks, since one has unitary time development until the particle interacts with the apparatus, after which the particle state, x^+ or x^- , is correlated with the measurement

outcome state X^+ or X^- . However, \mathcal{G}_2 is only one of a collection of equally valid but mutually incompatible ways of using quantum mechanics to describe the measuring process. From the point of view of fundamental quantum theory there is no reason to prefer \mathcal{G}_2 to the unitary family \mathcal{G}_0 . To be sure, the latter cannot be used to describe the measurement outcome, for, as pointed out earlier, S does not commute with X^+ or X^- . Thus from a practical point of view \mathcal{G}_2 is more useful than \mathcal{G}_0 . But there is no reason to prefer \mathcal{G}_2 to \mathcal{G}_1 , and \mathcal{G}_1 has the advantage that it allows one to think of the measurement process as a *measurement* in the usual sense of that term: a procedure by which the macroscopic outcome reflects a property the measured system had *before* the measurement takes place. In practice, most measurements on microscopic quantum systems carried out in the laboratory can best be thought of using a viewpoint akin to that of \mathcal{G}_1 : a gamma ray is detected by destroying it, the momentum of a charged particles emerging from a collision vertex is measured by changing it in a magnetic field, etc. (In these cases (23) is not an appropriate model, because the measurements are destructive, but the histories approach handles these equally well, Ch. 17 of [35], and shows that the measurement outcomes are correlated with quantum states which existed before the measurement interaction.) Descriptions analogous to \mathcal{G}_2 play very little role in physics apart from their appearance in textbook lists of quantum axioms where they have confused generations of students, not because they are wrong, but because the corresponding “wave function collapse” has been misinterpreted as a physical phenomenon, rather than just one of many ways of describing quantum time development. Rectifying that misinterpretation is, as we shall see, the key to untangling several relativistic quantum paradoxes.

III Relativistic Quantum Histories

III A Kinematics and dynamics

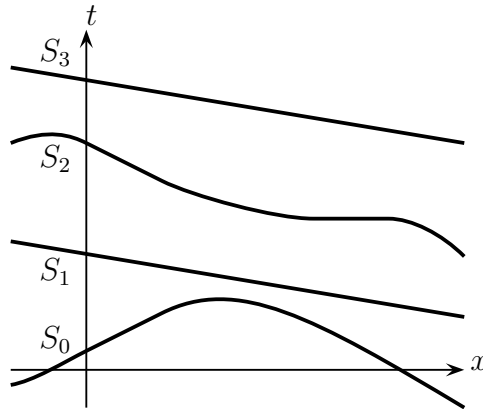


Figure 1: A possible collection of time-ordered spacelike hypersurfaces.

A plausible generalization of the histories approach described in Sec. II can be carried

out in the following way. Introduce a collection $\{S_j\}$, $j = 0, 1, 2, \dots$, of smooth, infinite, nonintersecting three-dimensional spacelike hypersurfaces, as suggested by the diagram in Fig. 1. They do not have to be “flat” hyperplanes, but the requirement that no two surfaces intersect means that if two or more hyperplanes belong to the collection, they must be parallel. As they do not intersect, the hypersurfaces can be ordered in time, and we assume that S_j is earlier than S_{j+1} , with S_0 the earliest hypersurface. (These spacelike surfaces have no thickness in the time direction, unlike the open regions in space-time employed by Hartle [40], Blencowe [41], and in algebraic quantum field theory [46]. This is consistent with our decision, Sec. I, to ignore problems of microlocality. Should it be necessary for technical mathematical reasons to introduce a small but finite thickness or duration in the time direction, that should not alter our conclusions.)

Next assume that for each hypersurface S_j there is a Hilbert space \mathcal{H}_j with identity operator I_j . Given a decomposition (3) of I_j in projectors, one can define histories of the form (2) on the history Hilbert space (5). The dynamical laws can be expressed using a collection of time development operators $\{T_{jk}\}$, where T_{jk} is a unitary map (bijective isometry) from \mathcal{H}_k onto \mathcal{H}_j , the analog of the nonrelativistic $T(t_j, t_k)$. The conditions analogous to (8) are, obviously,

$$T_{jj} = I_j, \quad T_{ij}T_{jk} = T_{ik}, \quad T_{jk}^\dagger = T_{kj}. \quad (29)$$

At this point one could introduce chain operators of the form (9), but for our purposes it is more convenient to introduce Heisenberg projectors on the Hilbert space \mathcal{H}_r of a special *reference hypersurface* (or hyperplane) S_r . As we shall ascribe no *physical* significance to the Heisenberg projectors — they are only introduced as a convenience for mathematical calculations — the relationship between S_r and the collection $\{S_j\}$ is arbitrary; in particular S_r may intersect the other hypersurfaces, or it could be identical to one of them. By means of the unitary time development operators T_{jr} mapping \mathcal{H}_r to the other Hilbert spaces we define the Heisenberg operator

$$\hat{P}_j^{\alpha_j} = T_{rj}P_j^{\alpha_j}T_{jr}. \quad (30)$$

corresponding to the projector $P_j^{\alpha_j}$. Heisenberg chain operators mapping \mathcal{H}_r to itself are then defined as a product of Heisenberg projectors (15), and the weights and consistency conditions are expressed in terms of these chain operators using (16) and (17), with an appropriate definition of the operator inner product \langle, \rangle .

Defining \langle, \rangle in terms of the trace, as in (11), is only satisfactory if the trace exists, which need not be the case, since \mathcal{H}_r is infinite. There is no problem if all the histories we are interested in are of the form (4) with Ψ_0 a pure initial state or a projector onto a finite subspace of \mathcal{H}_0 . Alternatively, one can introduce a density operator ρ_0 (with unit trace) on \mathcal{H}_0 , define its Heisenberg counterpart $\hat{\rho} = T_{r0}\rho_0T_{0r}$ as in (30), and replace the operator inner product \langle, \rangle in (16) and (17) with

$$\langle A, B \rangle_\rho = \text{Tr}(\hat{\rho}A^\dagger B). \quad (31)$$

(In this case it is best to regard $\hat{\rho}$ as a pre-probability; see Sec. 15.2 of [35].) Of course, any other \mathcal{H}_j could be used in place of \mathcal{H}_0 , but physicists typically tend to employ an initial

condition (we live in a thermodynamically irreversible world). Given a family of histories satisfying the consistency conditions (17), its physical interpretation is precisely the same as in the nonrelativistic case: one and only one history belonging to the family actually occurs in any given situation. The probabilities of histories are determined by the weights and whatever constitutes one’s information about the initial state or experimental setup; see, e.g., Sec. 9.1 of [35].

Since histories with events on a finite set of spacelike hypersurfaces may seem odd to a reader accustomed to the continuous time trajectories familiar in classical physics, the following comments may be helpful. Just as in the nonrelativistic case — see the discussion of refining a family in Sec. II C — it is always possible to introduce additional spacelike hypersurfaces between (or before or after) those in the collection $\{S_j\}$, and extend histories of the form (2), without changing their physical meaning, by introducing the trivial event I on these additional hypersurfaces. This shows that defining histories on a finite collection of hypersurfaces does not imply that the world ceases to exist at intermediate space-time points, it simply means that these histories contain no information about what is happening elsewhere than on these hypersurfaces. Think of being outside on a dark night during a thunder storm, when flashes of lightening illuminate the landscape at certain times, but nothing can be seen in the intervening intervals. In the nonrelativistic case one can, to be sure, produce histories which are described by non-trivial (not equal to I) projectors at all times, thus “filling in the gaps” in (2); one method of doing this is discussed in Sec. 11.7 of [35]. However, different ways of filling the gaps lead to incompatible families, and since there is no limit to the number of times which enter a discrete history of the form (2), there is really no need to fill the gaps from the point of view of providing an adequate physical description. The same comment applies in the relativistic case, at least for the purposes of the present paper.

Just as in Sec. II, a family of histories satisfying the consistency conditions will be called a *consistent family* or *framework*. A refinement \mathcal{F}' of a framework \mathcal{F} must include among its hypersurfaces $\{S'_k\}$ all the hypersurfaces associated with \mathcal{F} , and on the latter the decomposition of the identity used in \mathcal{F}' must be a refinement of the one used in \mathcal{F} . In order for it to be a framework, \mathcal{F}' must satisfy the consistency conditions. Two frameworks \mathcal{F} and \mathcal{G} will be said to be *compatible* provided they possess a common refinement which is itself a framework; otherwise they are *incompatible*. This is the same definition employed in the nonrelativistic case. The *single framework rule* is also the same as for nonrelativistic quantum theory: quantum descriptions must always be constructed using a single framework. If two frameworks are not identical but are compatible, a common description can be constructed using their common refinement. However, descriptions corresponding to incompatible frameworks cannot be combined.

III B Local regions and properties

For discussing macrolocality and quantum paradoxes we shall want to consider spacelike regions R_j of finite extent, see Fig. 2(a), each consisting of one or else a small number of connected pieces belonging to a spacelike hypersurface S_j , with each piece of “macroscopic”

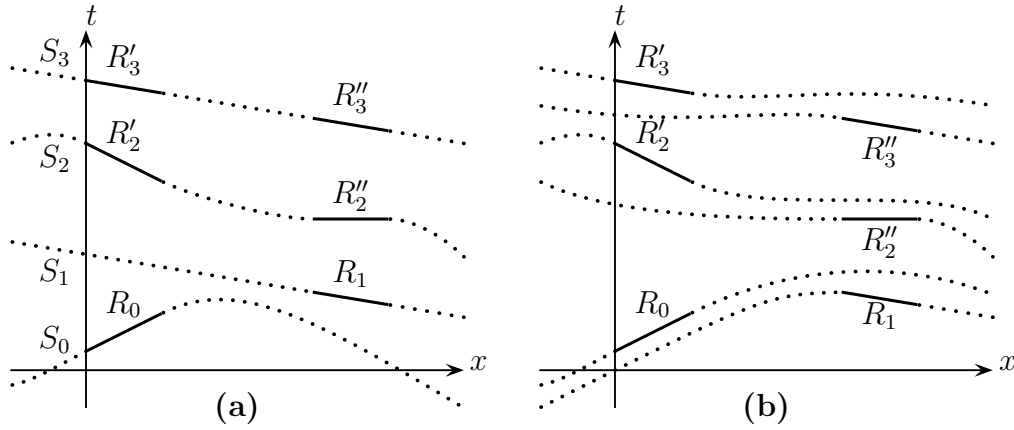


Figure 2: (a) Finite regions R_j , possibly consisting of more than one connected piece, belonging to infinite spacelike hypersurfaces. (b) An alternative way of embedding the same finite regions in spacelike hypersurfaces.

size, much larger than a Compton wavelength, with “reasonable” (e.g., piecewise smooth) boundaries — imagine a sphere or a cube. (Much of the following discussion is valid if R_j is an infinite piece of S_j , but we shall be interested in cases in which it is finite.) By making each R_j part of some S_j , with the collection $\{S_j\}$ satisfying the conditions given in Sec. III A, we ensure that it is possible to impose a well-defined *time ordering* on these finite regions. One could also do this by working out a set of conditions applicable directly to the collection $\{R_j\}$, but requiring that they be embeddable in a time ordered collection of infinite hypersurfaces is a fairly simply and intuitive way to proceed.

The lowbrow way to think of a property P_j as local to or localized in R_j is to imagine that the Hilbert space \mathcal{H}_j associated with S_j is a tensor product $\mathcal{H}_j^r \otimes \mathcal{H}_j^s$, with \mathcal{H}_j^r associated with R_j and \mathcal{H}_j^s associated with the complement of R_j in S_j . Then suppose that on this tensor product P_j is of the form $P_j^r \otimes I_j^s$, with I_j^s the identity on \mathcal{H}_j^s . Thus P_j^r , an operator on \mathcal{H}_j^r , tells one something about the state of affairs inside R_j , while I_j^s is totally uninformative about what is going on elsewhere. Note that the usual physicists’ convention allows the same symbol P_j to represent P_j^r or $P_j^r \otimes I_j^s$, without (much) risk of confusion, and we shall make use of this liberty. The highbrow way of thinking about a localized property requires dealing seriously with the microlocality problem, see Sec. I, and is outside the scope of the present paper. If R_j can be embedded in two different spacelike hypersurfaces S_j and S'_j , then the same local event will be represented by two different projectors in the Hilbert spaces \mathcal{H}_j and \mathcal{H}'_j . We shall make the plausible assumption that these two projectors lead to one and the same Heisenberg projector when mapped via (30) to the reference space \mathcal{H}_r using the appropriate time development operators T_{rj} and T'_{rj} .

Next we make the very important assumption that the dynamics embodied in the collection of unitary time transformations T_{jk} is local in the sense that whenever R_j and R_k are two regions which are spacelike separated (i.e., each point in R_j is at a positive spacelike separation from each point in R_k), and P_j and Q_k are projectors referring to physical events

(or properties) in R_j and R_k , respectively, the corresponding Heisenberg operators commute:

$$\hat{P}_j \hat{Q}_k = \hat{Q}_k \hat{P}_j. \quad (32)$$

This is often referred to as the principle of causality [46]. For our analysis it has the important consequence that in cases in which more than one time ordering is possible for a collection of regions $\{R_j\}$, because some of the regions are spacelike with respect to each other — for example, R_0 and R_1 in Fig. 2 — these different time orderings will give rise to the same chain operators (15), since the corresponding Heisenberg operators commute with each other.

Suppose that R_j consists of two or more disconnected subregions, e.g., R_3 in Fig. 2(a). We shall say that a projector P_j which is local to R_j is in addition *localized with respect to these subregions* if it is a product of projectors, one (possibly the identity) for each subregion, i.e., local to this subregion. Otherwise P_j is *entangled* with respect to these subregions. The distinction is important, because, as we shall see later, one may wish to embed the subregions in distinct nonintersecting hypersurfaces, as in Fig. 2(b), which are part of a time-ordered collection. If P_j is localized, this construction causes no difficulty, because each of the factors making up P_j is itself a local projector on the corresponding subregion, and the physical interpretation of this projector does not depend on the hypersurface in which the subregion is embedded. But if P_j is entangled, one cannot change the embedding by placing the subregions (at least those among which P_j is entangled) in distinct hypersurfaces without violating the condition, fundamental to our construction of relativistic histories, that each projector representing a single event in a history be associated with a particular hypersurface in a time-ordered collection of such surfaces. This difference between localized and entangled projectors will play a significant role in the later discussion of quantum paradoxes.

III C Lorentz invariance

Lorentz invariance requires that the “laws of physics” be the same in every Lorentz frame. In the preceding analysis the whole discussion has been carried out for a single Lorentz frame, let us call it \mathcal{L} . What should we expect if we use a different Lorentz frame \mathcal{L}' , thought of as a different choice for a coordinate system?

Each spacelike hypersurface S_j should be thought of as consisting of a definite collection of space-time points which is unchanged when the new coordinate system \mathcal{L}' is adopted. All that happens is that the quartet of numbers $r = (t, x, y, z)$ representing a particular space-time point is replaced by a new quartet $r' = (t', x', y', z')$. The symbol S'_j can be used to denote the same collection of space-time points as S_j , but relabeled using the new coordinates. If in \mathcal{L} the hypersurface S_j is specified by an equation

$$t = \tau_j(x, y, z), \quad (33)$$

then in \mathcal{L}' the same hypersurface, denoted by S'_j , will be specified in the same manner, by setting t' equal to a different function $\tau'_j(x', y', z')$.

Let us assume that there are well-defined rules based upon the function τ for assigning a Hilbert \mathcal{H}_j to the surface S_j , and that these rules do not depend upon the Lorentz frame. Of

course they will assign a different Hilbert space \mathcal{H}'_j to S'_j because τ' is not the same function as τ . However, we can expect that \mathcal{H}'_j is related to \mathcal{H}_j by a unitary map (bijective isometry) L_j which carries some $|\psi\rangle$ in \mathcal{H}_j onto a $|\psi'\rangle$ in \mathcal{H}'_j representing the same physical property. Next assume that the unitary time transformation T_{jk} mapping S_k to S_j is determined in a unique way by the two functions τ_j and τ_k , by rules which do not depend upon the Lorentz frame once these functions are given. In the same way, T'_{jk} mapping S'_k to S'_j will be determined by the functions τ'_j and τ'_k . The Lorentz invariance of the dynamics is then expressed by the requirement

$$T'_{jk} = L_j T_{jk} L_k^\dagger \quad (34)$$

for every pair j and k .

A history embodying the same physical events as in (2) will when expressed using the Hilbert spaces \mathcal{H}'_j be of the form

$$Y'^\alpha = P_0'^{\alpha_0} \odot P_1'^{\alpha_1} \odot P_2'^{\alpha_2} \odot \cdots \odot P_f'^{\alpha_f}, \quad (35)$$

with

$$P_j'^{\alpha_j} = L_j P_j^{\alpha_j} L_j^\dagger. \quad (36)$$

It is then easy to show that the weights calculated using the chain operators (15) for such histories in \mathcal{L}' are the same as their counterparts in \mathcal{L} , and the consistency conditions (17) hold in \mathcal{L}' if and only if they hold in \mathcal{L} , using the operator inner product defined in (11), or the one in (31), provided $\hat{\rho}$ is replaced by a suitable $\hat{\rho}'$. Thus the descriptions in the two Lorentz frames are physically equivalent to each other. A final point has to do with locality and the condition (32) for Heisenberg operators associated with regions which are spacelike separated from each other. All one needs to note is that regions which are spacelike separated in one Lorentz frame are also spacelike separated in any other, and the transformation rules in (36) ensure that $\hat{P}_j \hat{Q}_k$ is identical to $\hat{Q}_k \hat{P}_j$ if and only if $\hat{P}'_j \hat{Q}'_k$ is the same as $\hat{Q}'_k \hat{P}'_j$.

To be sure, all the difficulties of Lorentz invariance have been “buried” in the assumption that appropriate transformations L_j exist, and that the unitary time transformations satisfy (34), whatever inertial frame \mathcal{L}' is employed. This, however, is as it should be: the present paper is not devoted to the difficult task of constructing a Lorentz-invariant relativistic theory. Instead, its purpose is to show how various quantum paradoxes are to be resolved, by the appropriate use of histories, within the framework of such a theory, assuming it exists.

IV Wave Function Collapse

IV A Introduction

Imagine a particle emitted in a nuclear decay, moving outwards as a spherical wave packet. When detected by a detector some distance away, its wave function, according to textbook quantum theory, collapses instantaneously to zero everywhere outside the detector, since

that is where the particle is now located. This collapse helps explain why the particle cannot be detected later by a second detector located further from the original decay. But the notion of such a collapse has troubled many physicists ever since the earliest days of quantum theory [47]. It is troubling because, among other things, what is instantaneous in one Lorentz frame is not instantaneous in another, and therefore in some Lorentz frames the collapse will travel faster than the speed of light, or even backwards in time, placing the effect earlier than the cause. In addition, if after a suitable time the detector has *not* detected the particle, the probability increases that the particle will be detected by another detector located further away, unless this second detector is shadowed by the first, so even nondetection can alter (collapse?) the particle’s wave function. (This has led to the rather confusing idea of an “interaction-free” measurement; see [49] and pp. 495f of [48].)

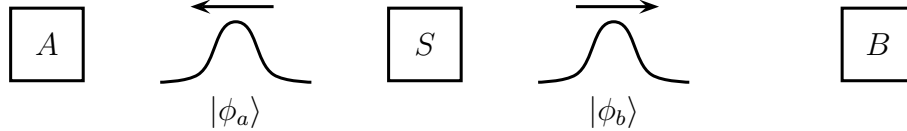


Figure 3: Wave packets representing a single particle, moving left and right from a source S towards detectors A and B .

In order to focus on essentials and simplify the discussion of how a histories approach resolves (or tames) these problems, it is useful to consider the analogous situation in one spatial dimension, as shown in Fig. 3, where the wave function of the particle (*one* particle, not two!) is given by a linear superposition

$$|\psi(t)\rangle = (|\phi_a(t)\rangle + |\phi_b(t)\rangle)/\sqrt{2} \quad (37)$$

of two wave packets moving outwards from a central source S towards two detectors A and B , with A closer to S than B . If A detects the particle, then at that instant of time (according to the collapse idea) the b part of the wave packet in (37) vanishes, whereas if A does *not* detect the particle, the superposition (37) is to be instantly replaced by $|\phi_b(t)\rangle$. Figure 3 is only schematic; we are interested in situations in which the distances separating source and detectors are very much larger than the widths of the wave packets, perhaps large enough that it takes light a significant amount of time to travel from S to A or B [50].

IV B Without detectors

As in Sec. II D, it is helpful to begin our analysis by considering a situation in which there are no detectors present. The dashed lines in Fig. 4(a) represent the centers of the wave packets $|\phi_a(t)\rangle$ and $|\phi_b(t)\rangle$ in the Lorentz frame \mathcal{L} where their velocities are equal and opposite. Unitary time development then corresponds to a family \mathcal{F}_0 with support (as defined in Sec. II D) consisting of the single history

$$\mathcal{F}_0: \psi(t_0) \odot \psi(t_1) \odot \psi(t_2) \odot \cdots, \quad (38)$$

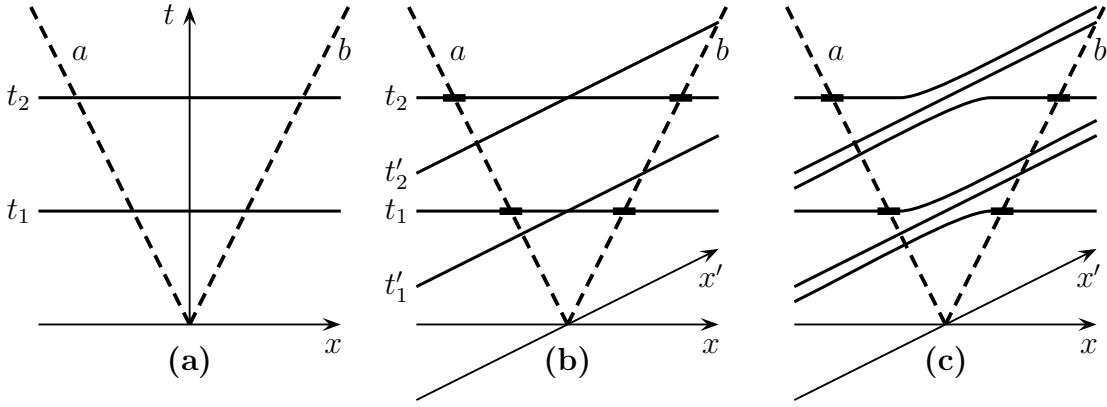


Figure 4: (a) Wave packet trajectories (dashed) and constant t lines in the \mathcal{L} space-time diagram. (b) Additional constant t' lines for Lorentz frame \mathcal{L}' . (c) Alternative hypersurfaces replacing the constant t lines.

where $\psi(t)$ is the projector onto $|\psi(t)\rangle$. To discuss the location of the particle, and in particular whether it is to the left or to the right of the source, we introduce at time t_j a decomposition of the identity

$$I_j = \sum_{\lambda_j} P_j^{\lambda_j}, \quad (39)$$

where the projectors $P_j^{\lambda_j}$ project onto nonoverlapping intervals of the x axis chosen so that they are large in comparison to the widths of the individual wave packets ϕ_a and ϕ_b , but small compared to the macroscopic length scales in Fig. 4. They are also chosen so that at each time t_j both $\phi_a(t_j)$ and $\phi_b(t_j)$ are well inside one of the intervals and not on the boundary between two of them. This way the family

$$\mathcal{F}_1: \quad \psi(t_0) \odot \{P_1^{\lambda_1}\} \odot \{P_2^{\lambda_2}\} \odot \{P_3^{\lambda_3}\} \odot \dots \quad (40)$$

will be consistent, and its support contains just two histories,

$$\mathcal{F}_1: \quad \psi(t_0) \odot \begin{cases} P_1^{a_1} \odot P_2^{a_2} \odot P_2^{a_3} \odot \dots, \\ P_1^{b_1} \odot P_2^{b_2} \odot P_2^{b_3} \odot \dots, \end{cases} \quad (41)$$

with equal weight. The first history says that as time increases the particle is in a series of intervals a_1, a_2, \dots falling along the dashed line a in Fig. 4(a), and the second that it is in a series of intervals falling along b . Thus they are coarse-grained quantum descriptions that approximate classical trajectories. The two histories are mutually exclusive possibilities: either the first occurs, so the particle follows trajectory a , or the second, so the particle follows b . The particle cannot follow both trajectories, or hop from one to the other. If at time t_2 it is, say, in the interval a_2 , then earlier it was in a_1 , and later it will be in a_3 .

The families \mathcal{F}_0 and \mathcal{F}_1 are incompatible, because $P_j^{a_j}$ and $P_j^{b_j}$ do not commute with $\psi(t_j)$, as follows from (37) and

$$P_j^{a_j}|\psi(t_j)\rangle = |\phi_a(t_j)\rangle/\sqrt{2}, \quad P_j^{b_j}|\psi(t_j)\rangle = |\phi_b(t_j)\rangle/\sqrt{2}. \quad (42)$$

To suppose that at t_j the particle is in the physical state $\psi(t_j)$ and that it is located in one of the two intervals $P_j^{a_j}$ or $P_j^{b_j}$ is as meaningless as saying that a spin-half particle is in the state $S_z = +1/2$, and at the same time ascribing to it values of S_x .

Next consider a family \mathcal{F}_2 with support consisting of

$$\mathcal{F}_2: \quad \psi(t_0) \odot \psi(t_1) \odot \begin{cases} P_2^{a_2} \odot P_3^{a_3} \odot \dots, \\ P_2^{b_2} \odot P_3^{b_3} \odot \dots. \end{cases} \quad (43)$$

Until t_1 the particle is in a nonlocal superposition, and thereafter it either follows the (coarse-grained) a trajectory or the b trajectory, two mutually exclusive possibilities, with probability $1/2$. One could, if one wants to, say that the initial description in terms of $\psi(t)$ “collapses” between t_1 and t_2 onto another sort of description in which the particle follows one of two distinct trajectories. However, one should not think of this “collapse” as a physical process. Instead it is the analog of a description of a spin-half particle in terms of S_z followed at a later time in terms of S_x , as in (22). The families \mathcal{F}_0 , \mathcal{F}_1 , and \mathcal{F}_2 are mutually incompatible in much the same way as their counterparts in Sec. II D. Each is a valid way of describing the quantum particle, and there is no “law of nature” that specifies that one of them is the “correct” description. However, there is a law of mathematics which prevents one from combining them, since there is no way of representing in the quantum Hilbert space a combination of events corresponding to noncommuting projectors.

There are always many incompatible ways of describing a quantum system, and the choice among them depends on what one wants to discuss. The use of \mathcal{F}_2 makes it possible to ascribe at time t_1 a relative phase to the sum of the wave packets making up $|\psi\rangle$, in the sense that a $+$ sign occurs rather than a $-$ sign on the right side of (37), but does not allow one to assign a position to the particle, even to the extent of saying that it is to the right or to the left of the source S . Assigning a coarse-grained position at this time requires that one use \mathcal{F}_1 , or something like it, in which case the relative phase of the wave packets becomes a meaningless concept. Incidentally, once the “split” has occurred in the family \mathcal{F}_2 the histories cannot be “joined” at a later time: replacing $P_3^{a_3}$ and $P_3^{b_3}$ in (43) with $\psi(t_3)$ violates the consistency conditions, and the same comment applies to \mathcal{F}_1 . In these respects the situation is analogous to that of the spin-half particle considered in Sec. II D.

IV C Different Lorentz frames

Consider a Lorentz frame \mathcal{L}' moving with respect to the frame \mathcal{L} we have employed thus far, with constant-time surfaces shown in Fig. 4(b) superimposed on the space-time diagram of (a). Let

$$|\psi'(t')\rangle = (|\phi'_a(t')\rangle + |\phi'_b(t')\rangle)/\sqrt{2} \quad (44)$$

represent the wave function as it develops unitarily in time in the new Hilbert space. The obvious analogs \mathcal{F}'_0 , \mathcal{F}'_1 , and \mathcal{F}'_2 of the families considered previously can be obtained by adding primes to the appropriate symbols in (38), (41), and (43), and the remarks made above about the physical interpretations of the \mathcal{F}_j apply equally to the \mathcal{F}'_j .

The three families \mathcal{F}_0 , \mathcal{F}'_1 and \mathcal{F}'_2 are not only incompatible with one another, each is also incompatible with each of the three families \mathcal{F}_0 , \mathcal{F}_1 , and \mathcal{F}_2 , because the constant-time hyperplanes of \mathcal{L}' intersect those of \mathcal{L} , and there is no way of placing them in a time-ordered sequence. However, the incompatibility of \mathcal{F}_1 and \mathcal{F}'_1 is only apparent, and can be removed by employing the “trick” shown in Fig. 4(c). Here the finite regions, shown with heavy lines, where the particle can be located at t_1 and t_2 in \mathcal{F}_1 have been embedded into an alternative set of hypersurfaces which do not intersect, and are thus compatible with, the hyperplanes used in \mathcal{F}'_1 . This construction is possible, as indicated at the end of Sec. III B, provided we are interested in properties which are localized in the separate subregions, rather than entangled among them. In the family \mathcal{F}_1 we are concerned with local properties: whether the particle is located in the a subregion or in the b subregion, rather than $\psi(t_1)$ and $\psi(t_2)$, which occur in the unitary family \mathcal{F}_0 , and are entangled between the a and the b subregions.

But does $P_1^{a_1}$, whose physical interpretation is that “the particle is in the (small) interval a_1 ”, really represent a *local* property? There is a subtlety here, for in the lowbrow approach outlined in Sec. III B a local property is represented by a projector on the Hilbert space of a (macro)local region, times the identity on another Hilbert space for the rest of the universe. In a Hilbert space of one-particle wave packets, $P_1^{a_1}$ is not of this form, because it tells us both that the particle is in a_1 *and* that it is not in some distant region; i.e., this projector provides more than local information. The way to get around this is to employ a many-particle Hilbert space, define $P_1^{a_1}$ to be the projector that tells us there is exactly one particle in a_1 , and use the initial state $\psi(t_0)$ to specify that the universe contains only one particle, as well as giving the wave packet for this one particle. In a history whose initial state is $\psi(t_0)$, and given a dynamical law that the particle cannot disappear or other particles appear, the event $P_1^{a_1}$ will allow us to infer that the particle is in a_1 and therefore not elsewhere, even though the projector $P_1^{a_1}$ by itself provides only local information. The reader for whom this argument is unnecessary should ignore it, while he who finds it inadequate is invited to construct a better version.

We conclude that in terms of their actual physical contents, \mathcal{F}_1 and \mathcal{F}'_1 are compatible, with a common refinement, call it \mathcal{F}_1^* , that uses the time ordering associated with the collection of hypersurfaces in Fig. 4(c). The support of \mathcal{F}_1^* again consists of two histories, one with the particle following trajectory a in a coarse-grained sense, described sometimes by an \mathcal{L} and sometimes by an \mathcal{L}' projector, and the other following trajectory b in a similar fashion. Aside from the subtleties associated with coarse graining, in both space and time, the trajectories agree with the picture provided by classical physics, even though they arise from a fully quantum-mechanical description.

On the other hand, the trick just discussed cannot be used in order to combine \mathcal{F}_1 with \mathcal{F}'_2 . While one can introduce a common set of hypersurfaces as in Fig. 4(c), the common refinement will not satisfy the consistency conditions. The trouble is that the particle can be localized on the b trajectory at t_1 in \mathcal{F}_1 , and this precedes (in the time ordering of the

hypersurfaces) the entangled state between a and b at t'_1 in \mathcal{F}'_2 . As noted above, trying to “uncollapse” a quantum description in this manner violates the consistency requirements. It is like introducing an S_z description into (21) after an S_x description has appeared. In addition, one cannot combine \mathcal{F}_2 with \mathcal{F}'_2 , for in this case the events at t_1 in the former and at t'_1 in the latter are both entangled, so the collection of hypersurfaces in Fig. 4(c) is no longer of any use. (It is possible, see the comments near the beginning of Sec. VII A, that some consistent generalization of the rules given in Sec. III might allow one to construct a common refinement in this case, but possible extensions of these rules fall outside the scope of the present paper.)

IV D Detectors

Most of the tools required to resolve (or tame) the paradox of wave function collapse are now in hand; all that remains is to introduce measurements. This we do using a fully quantum-mechanical description of the two particle detectors shown in Fig. 3. Let their states when ready to detect a particle be denoted by $|A(t)\rangle$ and $|B(t)\rangle$, respectively, and suppose that the detection event for the particle when represented by wave packet $|\phi_a\rangle$ is given by a unitary time development

$$|\phi_a\rangle|A\rangle \rightarrow |A^*\rangle. \quad (45)$$

Here $|A^*\rangle$ is a state in which this detector has detected the particle, as indicated by the position of a large pointer, or some other macroscopic change that clearly distinguishes it from the untriggered or ready state $|A\rangle$. The time arguments have been omitted in (45); one should think of the left side as at a time t' before the particle interacts with the detector, while the right side as at a time t'' after the interaction, when the particle is trapped inside the detector. If, on the other hand, the particle is represented by wave packet $|\phi_b\rangle$, it will not interact with detector A , and the counterpart of (45) is

$$|\phi_b\rangle|A\rangle \rightarrow |\phi_b\rangle|A\rangle. \quad (46)$$

The analogous expressions for the B detector are:

$$|\phi_a\rangle|B\rangle \rightarrow |\phi_a\rangle|B\rangle, \quad |\phi_b\rangle|B\rangle \rightarrow |B^*\rangle. \quad (47)$$

With both detectors initially in the ready state, the overall unitary time development corresponding to the space-time diagram in Fig. 5(a) is represented by a family with support

$$\mathcal{G}_0: \quad \Psi_0 \odot \Psi_1 \odot \Psi_2 \odot \Psi_3 \odot \cdots, \quad (48)$$

where

$$\begin{aligned} |\Psi_0\rangle &= |\psi(t_0)\rangle|A\rangle|B\rangle \\ |\Psi_1\rangle &= |\psi(t_1)\rangle|A\rangle|B\rangle \\ |\Psi_2\rangle &= (|A^*\rangle + |\phi_b(t_2)|A\rangle)|B\rangle/\sqrt{2} \\ |\Psi_3\rangle &= (|A^*\rangle|B\rangle + |A\rangle|B^*\rangle)/\sqrt{2}, \end{aligned} \quad (49)$$

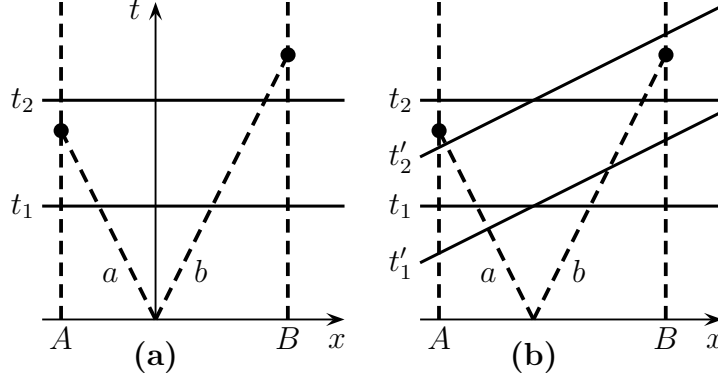


Figure 5: (a) Wave packet trajectories (sloping dashed lines) and detector trajectories (vertical dashed lines) in the \mathcal{L} space-time diagram. (b) Additional constant t' lines for Lorentz frame \mathcal{L}' .

and time arguments have again been omitted from the detector states. Note that since the detectors are macroscopic objects, both $|\Psi_2\rangle$ and $|\Psi_3\rangle$ are examples of MQS states; see (26) and the comments following it.

The family \mathcal{G}_1 with support

$$\mathcal{G}_1: \quad \Psi_0 \odot \left\{ \begin{array}{l} P_1^{a_1} AB \odot A^* B \odot A^* B \odot \dots \\ P_1^{b_1} AB \odot AB \odot AB^* \odot \dots \end{array} \right. \quad (50)$$

is analogous to \mathcal{F}_1 in Sec. IV B. In the first history the particle follows the a trajectory and is detected by A , while the B detector is unaffected. In the second history it is the A detector that remains in its ready state while the particle moves along trajectory b and triggers B . Note, in particular, that from the fact that detector A triggers one can conclude that the particle was earlier moving towards this detector, rather than towards B , while if at t_2 detector A has not detected the particle, one can infer that the particle is (and was) moving towards detector B , and will later be detected by B . Such inferences are not at all mysterious, and make no reference to wave function collapse. Instead, they are consequences of the fact that the two histories in (50) are the only two possibilities; all others have zero probability. There are, to be sure, many other frameworks that can be used to describe this situation in quantum terms, but any framework that contains the events needed to draw the conclusions stated above will assign them the same probabilities as \mathcal{G}_1 ; see Sec. 16.3 of [35].

Another family \mathcal{G}_2 with support

$$\mathcal{G}_2: \quad \Psi_0 \odot \psi(t_1) AB \odot \left\{ \begin{array}{l} A^* B \odot A^* B \odot \dots \\ \phi_b(t_2) AB \odot AB^* \odot \dots \end{array} \right. \quad (51)$$

is analogous to \mathcal{F}_2 in Sec. IV B in that the particle remains in a superposition state ψ at time t_1 , whereas at t_2 there has been a “collapse” into two possibilities: either the particle has been detected by A , the first history, or, in the second history, it has not been detected by A

and is still on its way to towards B , which will have detected it by t_3 . (In the second history one could use the interval projector $P_2^{b_2}$ at time t_2 in place of the wave packet projector $\phi_b(t_2)$; for our purposes it makes no difference.) Note that just as the “collapse” in \mathcal{F}_2 is not a physical process, but represents a change in the type of description being employed, so also in \mathcal{G}_2 it is not something which is brought about by some “law of nature”, as is evident from the fact that \mathcal{G}_0 and \mathcal{G}_1 are equally good descriptions, and in neither of them does interaction with a measuring apparatus produce a corresponding “collapse.”

Introducing another description based on constant time (hyper)surfaces in a second Lorentz frame \mathcal{L}' , Fig. 5(b), leads to no new principles beyond those already discussed in Sec. IV C. There is a formal incompatibility between descriptions based upon constant t and constant t' hyperplanes, but if one is concerned with local properties it is possible to adopt a common refinement in which the particle either moves along the a trajectory to be detected by A , or along the b trajectory to be detected by B , and can be seen to do so using either \mathcal{L} or \mathcal{L}' projectors, provided that these descriptions are interleaved and one does not try and impose them simultaneously at the same (macro)point in space-time. Note, in particular, that if one is using such a local quantum description, the fact that in \mathcal{L}' the particle can reach B earlier than it can reach A , the reverse from \mathcal{L} , is no more paradoxical than in classical relativistic physics. It is only if one insists upon employing a collapse picture using \mathcal{G}_2 , (51), along with its counterpart \mathcal{G}'_2 in \mathcal{L}' , that difficulties arise. These two families are incompatible according to the rules of Sec. III, and it makes no sense to ask which of them is correct, or when it is that the collapse “really” occurs, etc.

IV E Summary

It is useful to summarize the lessons provided by the preceding analysis by restating its conclusions as they apply to the situation which initiated our discussion: a particle moving outwards in a spherical wave, which may later encounter a detector (or perhaps several detectors). The spherical wave corresponds to unitary time development (solving Schrödinger’s equation), and if unitary time development is applied to the full quantum system of particle plus detector, the result will be an MQS state of a triggered and untriggered detector. While this, the analog of \mathcal{G}_0 in (48), is a perfectly valid quantum description, it is not useful for answering questions such as: Did the detector detect the particle? Where was the particle before it was detected? Posing these questions requires using projectors which do not commute with the projector $\Psi(t)$ on the state $|\Psi(t)\rangle$ resulting from unitary time evolution, and hence they are meaningless within that framework. Instead, one must use a family of stochastic histories in which at an appropriate time the detector has or has not detected the particle, something analogous to \mathcal{G}_1 or \mathcal{G}_2 in (50) and (51). In families which are the analogs of \mathcal{G}_1 , the particle follows a coarse-grained trajectory, the quantum counterpart of a “classical” description, moving in a straight line from the source of the decay until it reaches (in some histories) or misses (in others) the detector. This is the type of description actually used by physicists when thinking about decays of unstable particles, especially when designing equipment with collimators and detectors, or considering sources of undesirable background (see, e.g., pp. 123f in [31]). Because the events in these families are *local* in

a coarse-grained sense, relative to macroscopic length scales, their behavior under Lorentz transformations is (essentially) the same as in classical relativistic physics.

Wave function collapse is *never* needed in order to produce physically-meaningful quantum descriptions, since one can always assign probabilities within a consistent family or framework using the Born rule and its consistent extension, and then use these to calculate appropriate conditional probabilities. There are, to be sure, families of histories, the analogs of \mathcal{G}_2 , which can be thought of as exhibiting a “collapse”. While these are perfectly legitimate quantum descriptions, the collapse can occur in the absence as well as in the presence of a measurement, and represents a change in the type of quantum description employed, not some sort of physical process. It is analogous to the physicist’s choice to describe an isolated spin-half particle during a certain time interval using S_z , and during a subsequent time interval using S_x , even though the unitary time development is trivial; see the comments following (21).

V EPR Paradox

V A Introduction

The celebrated Einstein-Podolsky-Rosen [13] or EPR paradox is usually discussed nowadays using the formulation introduced by Bohm [51] in which two spin-half particles a and b prepared in a spin-singlet state

$$|s_0\rangle = (|z_a^+\rangle|z_b^-\rangle - |z_a^-\rangle|z_b^+\rangle)/\sqrt{2}, \quad (52)$$

where $|z_a^+\rangle$ is the state $S_{az} = +1/2$ of particle a , etc., fly apart from each other, and the spin of one of the particles is later measured. If S_{az} is measured and the outcome is $+1/2$, this means that $S_{bz} = -1/2$ for particle b , while an outcome of $-1/2$ implies that $S_{bz} = +1/2$. Similarly, if S_{ax} is measured, then S_{bx} will have the opposite value: $S_{bx} = -S_{ax}$. The paradox is that one seems able to assign a value to either S_{bz} or to S_{bx} depending upon which measurement is carried out on particle a , and since the measurement should not influence particle b , this seems to mean that both S_{bz} and S_{bx} have well-defined values, contrary to the principles of quantum theory.

Neither the original EPR formulation nor that of Bohm make use of relativistic quantum theory. But the paradox becomes a bit sharper in a relativistic context, for particles a and b could be spacelike separated when a measurement is made on a , so that any influence on b would seem contrary to the principles of relativity theory. In addition, if the paradox is formulated in terms of wave function collapse — the spin state $|s_0\rangle$ changes instantly to either $|z_a^+\rangle|z_b^-\rangle$ or $|z_a^-\rangle|z_b^+\rangle$ when S_{az} is measured — one encounters the same problem noted in Sec. IV A: the collapse is not Lorentz invariant, as well as (or because of) being instantaneous between spacelike separated points.

Rather than a single measurement on particle a , one can imagine separate spin measurements on a and b , and if they are of the same component, say S_z , then they will always give opposite results, $S_{bz} = -S_{az}$. It is worth emphasizing that this sort of *correlation*, even when

the measurements are carried out in spacelike separated regions, is not in itself paradoxical, as can be seen from a simple classical example. A pair of opaque envelopes is prepared, one containing a red and the other a green slip of paper. One envelope, chosen at random, is taken by astronaut Alice on a voyage to Mars, while the other remains behind on the desk of Bob at mission control. By opening her envelope and observing (“measuring”) the color of the slip of paper, Alice at once knows the color of the slip of paper in Bob’s envelope, and thus the color that Bob will observe (or perhaps has already observed) when he opens it, even if that event occurs at a spacelike separation. As with every classical analogy, this one is not adequate for illustrating all aspects of the quantum situation, but it does help clarify what is and is not specific to quantum theory.

V B Without measurements

As in Sec. IV, we shall first analyze what happens in the absence of measurements, assuming the two-particle wave function satisfying Schrödinger’s equation is given at time t by

$$|\psi(t)\rangle = |\omega(t)\rangle|s_0\rangle, \quad |\omega(t)\rangle = |\phi_a(t)\rangle|\phi_b(t)\rangle, \quad (53)$$

where $|\phi_a(t)\rangle$ and $|\phi_b(t)\rangle$ are wave packets of the sort shown in Fig. 3, except that now they refer to *two distinct* (and distinguishable) particles. Their trajectories in a space-time diagram are shown by the dashed lines in Fig. 4, where once again we assume that the distances are macroscopic, much larger than the microscopic extent of a wave packet. Since we are interested in the spins rather than the positions of the particles, it is convenient to ignore the latter, and think of

$$\mathcal{F}_0: \quad \psi_0 \odot s_0 \odot s_0 \odot \cdots, \quad (54)$$

as a unitary history, with ψ_0 the projector on the initial state $|\psi(t_0)\rangle$ and s_0 on the spin singlet state $|s_0\rangle$. In this family nothing can be said about any component of the spin angular momentum of particle a or of particle b , since the projectors for individual spin states, such as $|z_a^+\rangle$, do not commute with s_0 .

More information about properties of individual spins is provided by the family

$$\mathcal{F}_1: \quad \psi_0 \odot \begin{cases} z_a^+ z_b^- \odot z_a^+ z_b^- \odot \cdots, \\ z_a^- z_b^+ \odot z_a^- z_b^+ \odot \cdots, \end{cases} \quad (55)$$

where each history occurs with probability $1/2$. The physical interpretation is straightforward: in the first history, particle a has $S_{az} = +1/2$ and particle b has $S_{bz} = -1/2$ at all times later than t_0 , whereas in the second history $S_{az} = -1/2$ and $S_{bz} = +1/2$. In either case the spins are opposite, $S_{bz} = -S_{az}$, in the same way as the colors of the slips of paper in the envelopes belonging to Alice and Bob.

Still another consistent family

$$\mathcal{F}_2: \quad \psi_0 \odot s_0 \odot \begin{cases} z_a^+ z_b^- \odot z_a^+ z_b^- \odot \cdots \\ z_a^- z_b^+ \odot z_a^- z_b^+ \odot \cdots, \end{cases} \quad (56)$$

is analogous to \mathcal{F}_2 in Sec. IV B: up to t_1 the spins are in the entangled singlet state, but thereafter they “collapse” into states in which each particle has a well-defined value of S_z . Of course this collapse, just like those discussed in Secs. II D and IV B, has nothing to do with any physical process, and instead reflects a change in the choice of basis in which to describe the spins of the two particles; the comments following (43) apply equally in the present case. The frameworks \mathcal{F}_0 , \mathcal{F}_1 , and \mathcal{F}_2 are mutually incompatible. In addition, consistency conditions mean that one cannot “uncollapse” the histories in \mathcal{F}_2 (or in \mathcal{F}_1) by replacing the S_z projectors at, say, t_3 with s_0 ; again, the situation is analogous to that discussed in Sec. IV B.

There is nothing special about the z direction. The family

$$\mathcal{F}_3: \quad \psi_0 \odot \begin{cases} x_a^+ x_b^- \odot x_a^+ x_b^- \odot \cdots \\ x_a^- x_b^+ \odot x_a^- x_b^+ \odot \cdots, \end{cases} \quad (57)$$

is as good a quantum description as \mathcal{F}_1 , and replacing z by x everywhere in (56) results in yet another consistent family. All of the frameworks discussed thus far are mutually incompatible, which does *not* mean that using one of them to construct a correct quantum description of the particle’s time evolution makes the others false, or that one must invoke some hitherto unknown law of nature to decide which framework is “correct.” Instead, think of each one as describing a somewhat different “aspect” of the time development of the quantum system, viewing it from a somewhat different perspective, and thus each framework allows one to answer a different set of physically sensible questions about the system. How are the values of S_{ax} and S_{bx} related to each other at some particular time? This can only be answered by employing a framework in which the relevant projectors occur at the time of interest; e.g., \mathcal{F}_3 must be used rather than \mathcal{F}_1 .

One does not have to use the same component of spin angular momentum for particles a and b . In the framework

$$\mathcal{F}_4: \quad \psi_0 \odot \begin{cases} z_a^+ x_b^+ \odot z_a^+ x_b^+ \odot \cdots, \\ z_a^+ x_b^- \odot z_a^+ x_b^- \odot \cdots, \\ z_a^- x_b^+ \odot z_a^- x_b^+ \odot \cdots, \\ z_a^- x_b^- \odot z_a^- x_b^- \odot \cdots \end{cases} \quad (58)$$

the four histories occur with equal probability, and there is no correlation between S_{az} and S_{bx} . For additional comments on this and other examples, see Sec. 23.3 of [35].

V C Measurements

The spin measuring devices introduced in Sec. II D can also be employed in the present context if supplied with a subscript to indicate which particle is being measured. For example, the device to measure S_{az} has an initial state $|Z_a\rangle$, and we assume that the unitary time development when it interacts with particle a has the form

$$|z_a^+\rangle|Z_a\rangle \rightarrow |z_a^+\rangle|Z_a^+\rangle, \quad |z_a^-\rangle|Z_a\rangle \rightarrow |z_a^-\rangle|Z_a^-\rangle. \quad (59)$$

For an S_{ax} measurement replace Z with X and z with x . Nondestructive measurements are not essential, but they simplify drawing connections with traditional discussions using wave function collapse.

If we assume world lines as in Fig. 5(a), but with the B detector eliminated, unitary time development starting with an initial state

$$|\Psi_0\rangle = |\Psi(t_0)\rangle = |\omega(t_0)\rangle|s_0\rangle|Z_a\rangle, \quad (60)$$

in which the detector is ready to measure S_{az} , results in a succession of states

$$\begin{aligned} |\Psi(t_1)\rangle &= |\omega(t_1)\rangle|s_0\rangle|Z_a\rangle, \\ |\Psi(t_2)\rangle &= |\omega(t_2)\rangle(|z_a^+\rangle|z_b^-\rangle|Z_a^+\rangle - |z_a^-\rangle|z_b^+\rangle|Z_a^-\rangle)/\sqrt{2}, \end{aligned} \quad (61)$$

and so forth; for t_3 and all later times the spin and detector states are the same as for $|\Psi(t_2)\rangle$.

Now $|\Psi(t_2)\rangle$ is an MQS state, so that the unitary family that contains it, the analog of \mathcal{G}_0 in (48), cannot be used to discuss the outcomes of measurements. Instead, we need something like

$$\mathcal{G}_1: \quad \Psi_0 \odot \begin{cases} z_a^+ z_b^- Z_a \odot z_a^+ z_b^- Z_a^+ \odot z_a^+ z_b^- Z_a^+ \odot \cdots, \\ z_a^- z_b^+ Z_a \odot z_a^- z_b^+ Z_a^- \odot z_a^- z_b^+ Z_a^- \odot \cdots, \end{cases} \quad (62)$$

where the two histories occur with equal probability. In the first of these $S_{az} = +1/2$ and $S_{bz} = -1/2$ at times t_1 and later, and the measurement outcome is Z_a^+ at times t_2 and later, as one would expect, while in the other history $+$ and $-$ are interchanged. This family corresponds to the classical analogy introduced in Sec. V A, where astronaut Alice's opening the envelope and seeing a red (or green) slip of paper reveals a prior state of affairs, and enables her to conclude that the one in Bob's envelope is of the opposite color. Of course, this is not surprising given our earlier discussion of the family \mathcal{G}_1 in Sec. II D and \mathcal{G}_1 in Sec. IV D.

One can construct a family \mathcal{G}_2 , the analog of (51), in which the spin state in both histories is $|s_0\rangle$ at time t_1 and the “collapse” occurs in the same time step as the measurement:

$$\mathcal{G}_2: \quad \Psi_0 \odot s_0 Z_a \odot \begin{cases} z_a^+ z_b^- Z_a^+ \odot z_a^+ z_b^- Z_a^+ \odot \cdots, \\ z_a^- z_b^+ Z_a^- \odot z_a^- z_b^+ Z_a^- \odot \cdots, \end{cases} \quad (63)$$

An equally good family is

$$\mathcal{G}_4: \quad \Psi_0 \odot s_0 Z_a \odot \begin{cases} z_a^+ x_b^+ Z_a^+ \odot z_a^+ x_b^+ Z_a^+ \odot \cdots, \\ z_a^+ x_b^- Z_a^+ \odot z_a^+ x_b^- Z_a^+ \odot \cdots, \\ z_a^- x_b^+ Z_a^- \odot z_a^- x_b^+ Z_a^- \odot \cdots, \\ z_a^- x_b^- Z_a^- \odot z_a^- x_b^- Z_a^- \odot \cdots, \end{cases} \quad (64)$$

the measurement counterpart of \mathcal{F}_4 , where the collapse occurs at the same time, but now the properties of particle b are uncorrelated with those of particle a . The existence of frameworks such as \mathcal{G}_1 and \mathcal{G}_4 as alternatives to \mathcal{G}_2 helps prevent one from drawing the erroneous conclusion that a measurement carried out on particle a has some mysterious long-range influence on particle b .

V D Different Lorentz frames

Consider a Lorentz frame \mathcal{L}' moving with respect to the frame \mathcal{L} we have used up till now, with constant time (t') surfaces as shown in Fig. 4(b). Frameworks \mathcal{F}'_j analogous to the \mathcal{F}_j of Sec. V B can be defined by introducing primes on the appropriate symbols in (53) to (58), just as in Sec. IV C, and all comments made above on the physical interpretation of these families apply equally to these new descriptions. As in Sec. IV C, each \mathcal{F}'_k is incompatible with each \mathcal{F}_j according to the rules of Sec. III A, but in the case of \mathcal{F}'_1 and \mathcal{F}_1 , which refer to *local* properties, one can use the “trick” in Fig. 4(c) in order to produce a common refinement which includes the events of both frameworks for all t_j and t'_j with $j > 0$, with either ψ_0 or ψ'_0 (choose one or the other) as the initial state. That is, the relative time ordering of events with spacelike separation is of no concern provided they are, indeed, spacelike separated and not represented by entangled projectors, such as s_0 .

There is, however, a complication not present in the earlier discussion in Sec. IV C, where we were only concerned with the presence or absence of a particle in some region of space. Here we are (at least potentially) interested in different properties, always of the same particle, represented by noncommuting projectors, such as S_{az} and S_{ax} . Suppose, for example, we are interested in intercalating into the two histories in \mathcal{F}_1 in (55) at some time between t_1 and t_2 a (local) property of particle a . If this is an \mathcal{L} event, in the sense of one defined using a projector on a hyperplane which is at a constant time in \mathcal{L} , then it must satisfy the consistency conditions; in particular, if it is a projector onto a spin state of particle a , it must be either z_a^+ or z_a^- . If, instead, we intercalate an \mathcal{L}' event, then it, too, must satisfy the consistency conditions. In either case these are determined, see (17), by modified Heisenberg chain operators in which the additional event is represented by its Heisenberg projector at an appropriate point in the defining product (15). That is to say, there are restrictions on which \mathcal{L}' properties can be consistently incorporated into an \mathcal{L} history, but they are of precisely the same form governing the addition of \mathcal{L} events to that history. While relativity theory adds technical complications, the basic rules for consistency are exactly the same as in nonrelativistic quantum theory.

When one is interested in *nonlocal* properties represented by projectors on entangled states between particles a and b , then, as noted in Sec. IV C, the “trick” of introducing new hypersurfaces, Fig. 4(c), will not work, and one must pay attention to the rules of Sec. III A in order to avoid a situation in which one entangled state in \mathcal{L}' “occurs” both before (for particle a) and after (for particle b) another (entangled or product) state in \mathcal{L} . It is meaningless to combine two such descriptions, in the precise sense that the theory as formulated in Sec. III cannot assign a meaning to the combination, even though the individual events are themselves parts of sensible quantum descriptions.

Including measuring apparatus in the discussion leads to nothing new beyond what has already been noted at the end of Sec. IV D. In particular, if a measurement outcome is being used to infer a property of some particle in a localized region, such an inference is possible whether or not the particle is moving relative to the measuring apparatus. Of course, if one is interested in a property of the particle in its own rest frame, this must be appropriately related to the frame in which the calculation is carried out. Such transformations, and their

analogues in classical relativistic physics, are not trivial, but these are technical issues not directly connected with the paradoxes associated with wave function collapse. The latter are best disposed of by abandoning the notion of collapse, at least as some sort of physical process, and instead using appropriate conditional probabilities based upon histories.

VI Hardy's Paradox

VI A Statement of the paradox

Hardy's paradox [21] resembles the EPR paradox in that it involves two well-separated particles in an entangled state. However, it is more striking in that certain assumptions, including Lorentz invariance, seem to lead to a contradiction: something is shown to be true that is known to be false. As well as the relativistic paradox discussed here, Hardy's original paper contains a slightly different paradox whose discussion requires the use of counterfactuals, and for that reason lies outside the scope of the present paper. Our exposition differs in some unimportant ways from Hardy's original, and makes use of the nonrelativistic analysis in Ch. 25 of [35] (which also discusses the counterfactual paradox).

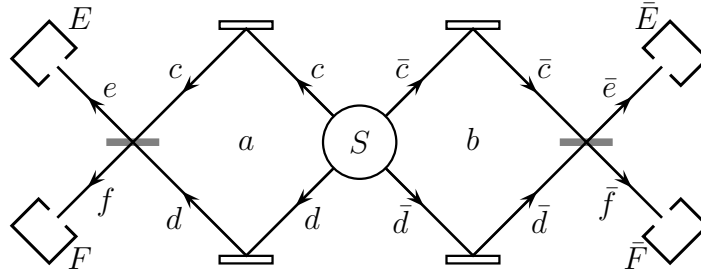


Figure 6: Apparatus for Hardy's paradox; see text.

Imagine a source S , Fig. 6, that simultaneously emits two particles a and b into the arms of two interferometers, in an initial state

$$|\psi_0\rangle = (|c\bar{c}\rangle + |cd\rangle + |d\bar{c}\rangle)/\sqrt{3}, \quad (65)$$

where $|c\bar{c}\rangle$ denotes a state in which particle a is moving through the c arm of its interferometer on the left side of the figure, and b is moving through the \bar{c} arm of the interferometer on the right. Note that (65) has no $|dd\rangle$ term, so it is never the case that a is in the d arm at the same time that b is in the \bar{d} arm.

The two beam splitters give rise to unitary time transformations

$$|c\rangle \rightarrow (|e\rangle + |f\rangle)/\sqrt{2}, \quad |d\rangle \rightarrow (-|e\rangle + |f\rangle)/\sqrt{2}, \quad (66)$$

$$|\bar{c}\rangle \rightarrow (|\bar{e}\rangle + |\bar{f}\rangle)/\sqrt{2}, \quad |\bar{d}\rangle \rightarrow (-|\bar{e}\rangle + |\bar{f}\rangle)/\sqrt{2}, \quad (67)$$

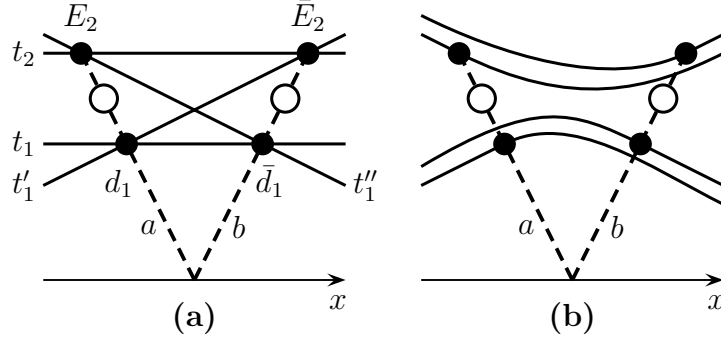


Figure 7: Space-time diagram for Hardy's paradox. The open circles represent the points where the particles pass through the beam splitters, the solid circles at t_2 (in \mathcal{L}) represent measurements which in \mathcal{L}' and \mathcal{L}'' are simultaneous with the corresponding points at t_1 . In (b) these points are on nonintersecting hypersurfaces.

where for convenience we have chosen real phases (unlike [21]). Unitary time development results in a state

$$|\psi_2\rangle = (-|e\bar{e}\rangle + |e\bar{f}\rangle + |f\bar{e}\rangle + 3|f\bar{f}\rangle)/\sqrt{12}, \quad (68)$$

at a time t_2 , Fig. 7(a), when both particles have passed through the beam splitters. Note that $|e\bar{e}\rangle$ occurs with a finite amplitude, implying that a and b will be simultaneously detected by E and \bar{E} with a probability of $1/12$.

If the interferometers are sufficiently large there will be a Lorentz frame \mathcal{L}' in which particle b is detected by \bar{E} or \bar{F} before particle a has reached the beam splitter on the left. It is then plausible that just before detection occurs at the time t'_1 in \mathcal{L}' , see Fig. 7(a), the wave function for the two particles is obtained by applying (67) but not (66) to (65), with the result

$$|\psi'_1\rangle = (2|c'\bar{f}'\rangle + |d'\bar{f}'\rangle + |d'\bar{e}'\rangle)/\sqrt{6}, \quad (69)$$

where the primes indicate wave packets at constant time in \mathcal{L}' . From this one can infer (e.g., by collapsing $|\psi'_1\rangle$ to $|d'\bar{e}'\rangle$) that if b is detected by \bar{E} , then at t'_1 particle a is in the d arm of its interferometer. Similarly, there will be a Lorentz frame \mathcal{L}'' in which a is detected by E or F before b reaches its beam splitter, and the counterpart of (69) is

$$|\psi''_1\rangle = (2|f''\bar{e}''\rangle + |f''\bar{d}''\rangle + |e''\bar{d}''\rangle)/\sqrt{6}. \quad (70)$$

From this it follows that if particle a is detected by E , then at t''_1 particle b is in the \bar{d} arm of its interferometer.

Next assume that the presence of particle a in arm d at a point on its trajectory indicated by d_1 in Fig. 7(a) does not depend upon whether one describes it using \mathcal{L} or \mathcal{L}' or \mathcal{L}'' , and that there is a similar invariance for particle b relative to arm \bar{d} , and for which of two detectors has detected a particle. Hardy calls this assumption the *Lorentz invariance of*

elements of reality, and it seems physically plausible, especially if one thinks of extremely large interferometers, so that the different Lorentz frames can be moving rather slowly with respect to each other. Assuming Lorentz invariance of this form, the inferences based on (69) and (70) can be transferred to the Lorentz frame \mathcal{L} , and one arrives at the disquieting conclusion that in those cases (occurring with probability 1/12) in which a and b are simultaneously (in \mathcal{L}) detected in E and \bar{E} at time t_2 , these particles were earlier, at t_1 , in the d and \bar{d} arms of their respective interferometers. But this conclusion is inconsistent with the initial state (65), since, as noted previously, it lacks a $|d\bar{d}\rangle$ component.

VI B Resolution of the paradox

It is helpful to analyze the logical structure of the argument leading to the paradox in a bit more detail. Using the space-time “points” (regions small compared to the distance between beam splitters) labeled in Fig. 7(a), the inferences based upon (69) and (70) can be written in the form

$$\bar{E}'_2 \Rightarrow d'_1, \quad E''_2 \Rightarrow \bar{d}''_1, \quad (71)$$

where \bar{E}'_2 means that in the Lorentz frame \mathcal{L}' particle b has been detected by \bar{E} at \mathcal{L} -time t_2 , an event which in \mathcal{L}' is simultaneous with the event d'_1 : particle a is in the d arm of its interferometer at \mathcal{L} -time t_1 . In a similar way, the “” events refer to \mathcal{L}'' . The assumption of Lorentz invariance of elements of reality implies that these inferences are still valid if we add or delete primes from any of the symbols in (71). Combining the two inferences with primes eliminated is what leads to the paradox. What can one say about this in terms of relativistic quantum histories?

The inferences in (71) refer to two hyperplanes which cross, and therefore combining them is a violation of the single family rule as formulated in Sec. III. But, as already noted in Secs. IV C and V D, one can get around this prohibition when considering local properties, such as those in (71), by the device of introducing curved hypersurfaces, as in Fig. 7(b). What is essential is the time order in which d_1 precedes E_2 , and \bar{d}_1 precedes \bar{E}_2 , which is true in any Lorentz frame (e.g., d''_1 precedes E''_2), while the relative temporal order of spacelike separated events, such as d_1 and \bar{E}_2 , is irrelevant, because we are not concerned with entangled states connecting two spacelike separated regions. To be sure, the wave functions in (69) and (70) are entangled states in the sense just mentioned. However, their only role in the argument is that they are a way of calculating (via wave function collapse) certain probabilities of local properties, probabilities which could be calculated just as well by other methods which make no reference to entangled states. In the terminology of Sec. 9.4 of [35], the entangled wave functions in (69) and (70) are pre-probabilities, and one need not think of them as representing physical reality. Thus each of the inferences in (71) can be justified by appeal to appropriate conditional probabilities, quite apart from the mathematical method used to calculate the probabilities.

Nevertheless, despite the fact that one is dealing with local properties and hence the crossing of hyperplanes is of no concern, one can only, as pointed out in Sec. V D, intercalate (local or nonlocal) events at additional times into a quantum history if the consistency

conditions are satisfied. This is a feature of both nonrelativistic and relativistic quantum theory, and in the present instance it prevents one from combining the two inferences in (71). Each of these inferences is valid by itself, in the sense that the events to the left and right of \Rightarrow can be placed in a consistent family that confirms the correctness of the inference through assigning a value of 1 to the corresponding conditional probability. However, the family required to justify the first inference is incompatible with that required to justify the second, and the two cannot be combined, as one would have to do to reach a contradiction.

To be more specific, any consistent history based on the initial state $|\psi_0\rangle$ of (65) which includes the event d_1 (or d'_1 or d''_1) cannot also include the later event E_2 (or E'_2 or E''_2). That is, it makes no sense to say that particle a is earlier in the d arm of its interferometer and later detected by E . And what is meaningless — an “element of unreality” — in one Lorentz frame is equally meaningless in another. Each inference in (71) refers to events which are spacelike separated, so their Heisenberg projectors commute, and for this reason they are compatible with the consistency conditions. However, the conclusion of the first inference is incompatible, in the quantum mechanical sense, with the premise of the second inference, so putting the two together is not possible, and this blocks the path to a logical contradiction.

In summary, Hardy’s relativistic paradox is resolved (or tamed) by paying careful attention to using rules of reasoning that are compatible with the mathematical structure of quantum theory. In particular, chaining arguments together in a manner which is perfectly acceptable in classical physics cannot be done in the quantum context without first checking that they belong to a single framework. That is the basic lesson to be learned from the Bell-Kochen-Specker result [52], and from the extensive discussion of quantum paradoxes in [35]. Indeed, the procedure used here for resolving the relativistic Hardy paradox is, in its essentials, identical to that used for its nonrelativistic counterpart in Sec. 25.3 of [35], to which the reader is referred for additional details, including detailed arguments for consistency and incompatibility of certain families.

By contrast (and contrary to the conclusion of Hardy’s original paper), the assumption of Lorentz invariance of local elements of reality gives rise to no problems: certain things one might expect to be the same in different Lorentz frames, such as the presence or absence of a particle, are indeed the same, or at least the assumption that this is true is not the origin of the paradox.

VII Summary and Conclusions

VII A Relativistic histories

The rules of nonrelativistic quantum kinematics summarized in Sec. II A have a straightforward generalization to the relativistic theory provided one adopts the condition, Sec. III A, that the spacelike hypersurfaces used to build a relativistic family of histories be time ordered, or, equivalently, cannot intersect each other. In the nonrelativistic theory the proper time ordering of events represented by Heisenberg projectors in the product (15) defining the chain operator is essential if one wants physically reasonable results, and this seems to demand nonintersecting hypersurfaces in the relativistic version, unless one wishes to con-

struct an entirely new theory. However, if the Heisenberg operators associated with two intersecting hypersurfaces commute with one another for the histories one is interested in, the chain operator will not depend upon their order. In particular, this is true if the Heisenberg operators are identical, and that suggests that combining certain unitary families (e.g., \mathcal{F}_0 and \mathcal{F}'_0 in Sec. IV C) may make sense. Whether an extension of the rules of Sec. III A allowing this sort of thing is worthwhile, and if so how best to formulate it, are open questions.

Locality and local properties are important concepts both for formulating and for resolving quantum paradoxes. The approach in Sec. III B seems adequate for the purposes of this paper, but could undoubtedly be improved, especially by making its technical assumptions more precise and less dependent on lowbrow intuition. Part of this task is to give a proper mathematical characterization of macrolocality, something which does not look trivial given the difficulties associated with microlocality, as mentioned in the introduction, though work by Omnès [39] may be pointing in the right direction. Another nontrivial task is that of constructing significant Lorentz-invariant theories satisfying the conditions stated in Sec. III C, in a way which can be applied to general hypersurfaces and not just to hyperplanes. The present paper contains nothing useful for this task, unless it be a clarification of what it is that one is after.

Such unresolved issues should not obscure the fact that the histories approach extends in a very natural way from nonrelativistic to relativistic quantum theory. The basic formulas defining histories, Heisenberg chain operators, weights or probabilities, and consistency conditions are formally the same in the nonrelativistic approach as summarized in Sec. II and in the relativistic extension in Sec. III. Not only are the symbols the same, the associated concepts are extremely close if not completely identical: the occurrence of events and histories, consistent families or frameworks, refinements, incompatible frameworks, the single framework rule, and probabilities. Even the examples are similar.

This close connection is hardly surprising given the fact, pointed out in the introduction, that relativistic versions of the histories approach have been around for some time. Nonetheless it is gratifying that some more recent developments first formulated in a nonrelativistic context — e.g., pre-probabilities and fully consistent schemes for assigning probabilities based on different sorts of data — can be “relativized” without any difficulty. This straightforward compatibility stands in marked contrast to the major difficulties which beset attempts to construct relativistic versions in some other “observer free” quantum interpretations [53,54].

VII B Resolving paradoxes

The three paradoxes resolved, or at least tamed, in Secs. IV, V and VI, are all connected with the idea that a measurement which takes place in some localized region can have effects at a distant place spacelike separated from the region in question. And they all invoke some form of wave function collapse in order to calculate probabilities or make inferences about the state of affairs at this distant place.

The basic strategy by which the histories approach disarms these paradoxes is by getting rid of wave function collapse. How to do this is shown in detail for the example considered in

Sec. IV; see the summary in Sec. IV E. The conclusion is that wave function collapse is not needed in quantum theory, and that if it is used it should *never* be thought of as a physical effect produced by a measurement. Because of its misleading connotations it might be best to get rid of wave function collapse altogether. There is nothing that can be calculated or (correctly) inferred using collapse which cannot be calculated or inferred equally well using conditional probabilities based on fundamental quantum principles that make no reference to measurements or to collapse. The histories approach can supply physical descriptions that resemble those of collapse (the \mathcal{G}_2 families of Secs. IV D or V C), and which help explain why the use of collapse as a calculational procedure yields correct answers. But it also supplies alternative descriptions (the \mathcal{F}_1 and \mathcal{G}_1 families in these same sections) which are much more useful for thinking about measurements from a physical point of view, because they show how a measurement outcome is related to some property of the microscopic system before the measurement took place. It is, in fact, this latter type of description that experimental physicists use for designing their equipment and analyzing their data. It is to be regretted that textbooks which include the rather unrealistic model of nondestructive measurements going back to von Neumann lack the basic concepts needed to understand, from a quantum perspective, the devices actually used in practice.

The use of families of the \mathcal{F}_1 or \mathcal{G}_1 type, with macrolocal properties both before as well as after a measurement (if any) takes place, has the further advantage that it simplifies the discussion of how the description of a quantum system must be altered in a relativistic theory if one uses a moving coordinate system. For families of this type, with an appropriate coarse graining in space and time, the quantum description becomes “classical” (as one would anticipate from the work of Gell-Mann and Hartle [28, 30]), and Lorentz transformations of particle trajectories behave the same way as in classical relativistic physics. States which are entangled over macroscopic distances, such as the pre-measurement properties in the \mathcal{G}_2 families, are not as easy to analyze, but the histories approach provides the tools needed for using entangled descriptions in a manner consistent with the basic principles of quantum theory, or combining entangled and local states at different times in histories in the same framework.

While there are many good reasons for removing wave function collapse from nonrelativistic quantum mechanics, the case is even stronger for a relativistic theory. The use of collapse understood as some sort of physical phenomenon is one of the main sources of the widespread notion that the quantum world is inhabited by superluminal influences, leading to a *prima facie* conflict with relativity theory. It is then necessary to prove theorems to the effect that these influences cannot carry information, i.e., they are completely unobservable phenomena. While a detailed discussion of the (supposed) nonlocality of quantum theory lies outside the scope of the present paper, it seems clear that to the extent that unobservable superluminal influences arise from thinking of wave function collapse as a physical phenomenon, disposing of the latter will get rid of the former. In any case, if collapse is not a physical phenomenon, discussions of when it actually occurs [2] are irrelevant to the physical theory.

Once wave function collapse is out of the way (or has been tamed, should one wish to continue using it), the resolution of the relativistic EPR and Hardy paradoxes is fairly

straightforward, using methods similar to those used employed for their nonrelativistic counterparts in Chs. 23 to 25 of [35]. As long as one limits oneself to a single framework, there is nothing paradoxical about EPR correlations in and of themselves, for they have a simple classical analog, Sec. V A. The notion that a measurement on particle a somehow influences particle b can be effectively undermined by noting some of the different frameworks that provide equally valid descriptions of the quantum time development. In the case of Hardy's relativistic paradox the source of the difficulty is not a failure of the Lorentz invariance of elements of reality, such as the presence or absence of a particle in a given region of space, but instead a process of reasoning which combines results from incompatible frameworks. In particular, the problem has to do with what one can meaningful say about the time dependence of the state of a *single* particle, rather than measurements on a second particle spacelike separated from the first, and thus relativistic considerations are actually irrelevant to the fundamental conceptual difficulty. Classical modes of reasoning easily give rise to contradictions if imported into the quantum domain without regard to the way in which the mathematics of quantum theory differs from that of classical physics.

We believe that the paradoxes considered in this paper are representative of a larger class, those in which traditional ideas of measurement and wave function collapse give rise to contradictions, or to nonlocal influences in apparent conflict with relativity theory. If that is true, then the methods used here for resolving the paradoxes of wave function collapse, EPR, and Hardy should work equally well for this larger collection, and help assuage the concern, seemingly widespread in the quantum foundations community, that quantum theory and relativity are fundamentally incompatible. The analysis in this paper indicates that the two go together very well when proper account is taken of the rules which are needed to make even nonrelativistic quantum mechanics a consistent theory.

There remain, of course, the problems of microlocality, understanding the quantum vacuum, constructing field theories using honest mathematics, and the like, whose resolution is not brought any nearer by anything in this paper. Unless it be indirectly through allowing a redirection of intellectual energy away from enigmas whose ultimate origin is the unsatisfactory manner in which probabilities have traditionally been introduced into quantum theory, both nonrelativistic and relativistic, and which disappear when this is done in a fully consistent way.

Acknowledgments

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